

Math 113 Homework 4
due Tuesday July 21, 2009

- (1) Q_8 , called the quaternion, is the set of size eight $\{\pm 1, \pm i, \pm j, \pm k\}$ where $(-1)x = -x$, $i^2 = j^2 = k^2 = -1$, and

$$\begin{array}{lll} i \cdot j = k & j \cdot k = i & k \cdot i = j \\ j \cdot i = -k & k \cdot j = -i & i \cdot k = -j \end{array}$$

You do not need to verify Q_8 is a group. (A presentation of Q_8 in terms of matrices is given on page 43.)

Do exercise 9.3 in Judson.

- (2) If n divides m , Find an injective homomorphism f from \mathbb{Z}_n to \mathbb{Z}_m . Prove that $\mathbb{Z}_m/\text{im}(f) \cong \mathbb{Z}_{m/n}$.
- (3) Do exercise 9.18 in Judson.
- (4) Find a surjective homomorphism from A_4 to \mathbb{Z}_3 .
- (5) Do exercise 9.20 in Judson.
- (6) Do exercise 9.23 in Judson.
- (7) Do exercise 9.26 parts (c) and (d) in Judson.
- (8) Do exercise 9.30 in Judson.