

Math 113 Homework 4  
due Wednesday July 15, 2009

- (1) Do exercise 4.29 in Judson.
- (2) Do exercise 5.10 in Judson.
- (3) Do exercise 5.19 in Judson.
- (4) Give an example of a group  $G$ , a subgroup  $H$ , and  $a, b \in G$ , so that  $aH = bH$  but  $Ha \neq Hb$ .
- (5) Do exercise 8.8 in Judson
- (6) (a) If  $G$  is a group of size 4, prove that  $G$  is isomorphic to either  $\mathbb{Z}_4$  or  $\mathbb{Z}_2 \times \mathbb{Z}_2$ .  
(b) If  $G$  is a group of size 6, prove that  $G$  is isomorphic to either  $\mathbb{Z}_6$  or  $S_3$ .
- (7) (a) Do exercise 8.24 in Judson.  
(b) Do exercise 8.25 in Judson.
- (8) For an abelian group  $G$ , we define  $\mathbb{Z}_2 \times G$  to be the set  $\{[0]_2, [1]_2\} \times G$  with the operation
$$([i]_2, x) \cdot ([j]_2, y) = ([i + j]_2, x^{(-1)^j} y)$$
  - (a) Prove that this operation is well defined and that  $\mathbb{Z}_2 \times G$  is a group.
  - (b) Prove that for  $n \geq 3$ ,  $\mathbb{Z}_2 \times \mathbb{Z}_n$  is isomorphic to  $D_n$ .
- (9) Do exercise 8.29 in Judson.
- (10) Let  $G$  be a group and let  $f : G \rightarrow G$  be the function  $f(x) = x^{-1}$ . Prove that  $f$  is an isomorphism if and only if  $G$  is abelian.