Math 113 Homework 3
due Tuesday July 7, 2009

(1) **More about Direct Products.** Recall from homework # 2 that if G and H are groups, then the direct product $G \times H$ is also a group. The operation on this group was $(g_1, h_1) *_{G\times H} (g_2, h_2) = (g_1 *_G g_2, h_1 *_H h_2)$.
  (a) Show that $\mathbb{Z}_2 \times \mathbb{Z}_2$ is not cyclic and that $\mathbb{Z}_2 \times \mathbb{Z}_3$ is cyclic.
  (b) Show that if A is a subgroup of G and B is a subgroup of H, then $A \times B$ is a subgroup of $G \times H$.
  (c) Give an example of groups G and H, and K a subgroup of $G \times H$ so that K is not the direct product of subgroups from each of G and H.
  (d) If L is a subgroup of $G \times H$, Define $\pi(L)$, the *projection* of L to G as

$$\pi(L) \equiv \{g \in G : (g, h) \in L \text{ for some } h \in H\}$$

  Prove the $\pi(L)$ is a subgroup of G.

(2) Do excercise 3.26 in Judson.
(3) Prove that if G is an abelian group, $a, b \in G$, and the orders of a and b are coprime, then the order of ab is the order of a times the order of b. Show that this can fail if G is not abelian.
(4) If n is a positive odd integer and z is a $(2n)$-th root of unity, prove that either z or $-z$ is an nth root of unity.
(5) Do excercise 3.44 in Judson.
(6) let $\omega = (1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14)$. For which integers i is ω^i a 14-cycle?
(7) Do excercise 4.13 in Judson.
(8) Do excercise 4.30 in Judson