Math 113 Homework 2
due July 1, 2009

(1) Do excercise 2.2 in Judson.
(2) Let $(J, \bullet)$ and $(K, \star)$ be groups. We can define a binary operation on $J \times K$, $\spadesuit : (J \times K) \times (J \times K) \to J \times K$ as
$$(j_1, k_1) \spadesuit (j_2, k_2) = (j_1 \bullet j_2, k_1 \star k_2)$$
Prove that $(J \times K, \spadesuit)$ is a group. This group is called the *direct product* of $(J, \bullet)$ and $(K, \star)$.
(3) Do excercise 2.24 in Judson.
(4) Do excercise 2.34 in Judson.
(5) If G is a group, H_1 and H_2 are both subgroups of G, and $H_1 \cup H_2 = G$, prove that $H_1 = G$ or $H_2 = G$. *(Hint: If this fails, how are H_1 and H_2 related?)*
(6) Do excercise 2.46 in Judson.
(7) Do excercise 2.52 in Judson.
(8) Suppose G is a nonempty finite set and $\star : G \times G \to G$ is an associative operation on G. Furthermore, assume that $\star$ is both left and right cancellative. (I.e if $a \star b = a \star c$ then $b = c$ and if $x \star y = z \star y$ then $x = z$.) Prove that $(G, \star)$ is a group. *(Hint: Things from a finite set eventually repeat.)*