Math 113 Homework 1
due June 25, 2009

(1) Do excercise 0.24 in Judson.
(2) (a) Show that the relation $\neq$ on the integers is symmetric, but neither reflexive or transitive.
    (b) Given an example of a relation which is symmetric and transitive, but not reflexive.
(3) Suppose X, Y, and Z are sets, and $f : X \to Y$, $g : X \to Y$, $h : Y \to Z$, and $i : Y \to Z$ are functions.
    (a) Prove that if h is injective and $h \circ f = h \circ g$, then $f = g$.
    (b) Prove that if f is surjective and $h \circ f = i \circ f$, then $h = i$.
(4) Given any function $g : X \to Y$, we can define a relation on X by

$$a \sim_g b \text{ if } g(a) = g(b)$$

    (a) Prove that $\sim_g$ is always an equivalence relation.
    (b) Given a set Z, and an equivalence relation $\bowtie$ on Z, we can define a function $f : Z \to$ equivalence classes of $\bowtie$ by

$$f(x) = [x]$$

        (i) Prove that f is actually a function.
        (ii) Prove that f is surjective.
        (iii) Prove that f is injective if and only if $\bowtie$ is equality.
        (iv) Prove that $\sim_f$ and $\bowtie$ are the same equivalence relation. So any equivalence relation comes from a function in this way.
(5) Prove by induction that $n^5 - n$ is always divisible by 5.
(6) Do excercise 1.12 in Judson.
(7) Let $p_1, \ldots, p_k$ be distinct primes, and $a_1, a_2, \ldots, a_k, b_1, b_2, \ldots, b_k$ be positive integers. Let $n = p_1^{a_1} \cdot p_2^{a_2} \cdot \ldots p_k^{a_k}$ and $m = p_1^{b_1} \cdot p_2^{b_2} \cdot \ldots p_k^{b_k}$. Computer $gcd(n, m)$.
(8) Do excercise 1.27 in Judson.
(9) Do excercise 1.30 in Judson.