Math 1B Practice Final # 2

Rob Bayer

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As always, this is WAY longer than the actual final will be and the inclusion or absence of certain topics should not be taken as an indication of the content of the actual final. As a group, decide what types of problems you're most worried about and try working through a few of those.

On the back of this worksheet is the formula sheet you will be given.

Integration

1. Find $\int \frac{\sqrt{x^2-1}}{x}$
2. Find $\int \frac{1}{x\sqrt{x-1}}$
3. Find the partial fraction decomposition for $\frac{x^3+2x}{x^3+1}$
4. Find the length of the curve $y = \frac{1}{x^2}$ between $x = 1$ and $x = 2$
5. Determine whether $\int_0^2 \frac{dx}{4x-5}$ is improper. Evaluate it, or show that it diverges.

Differential Equations

1. Find the general solution to $y' = \cos^2 y \sin^3 x \cos^2 x$
2. Solve $y' + (\cos x)y = \sin x \cos x$
3. Find the general solution to $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$
4. Solve the initial value problem $y'' + y = \sin x + e^x;\ y(0) = 0, y'(0) = 0$
5. Sketch a direction field for $y' = xy - 1$
6. Consider the differential equation $y'' + y = 0$
   (a) Even though it's the completely incorrect method to use, solve this using power series.
   (b) Now solve it the easy way. Do you get the same answer both ways?

Sequences and Series

1. Determine whether each of the following converge or diverge:
   (a) $\sum_{n=0}^{\infty} \frac{1000^n}{n!}$
   (b) $\sum_{n=10}^{\infty} \frac{1}{n(\ln(n))^{3/2}}$
   (c) $\sum_{n=1}^{\infty} \left(\sin\left(\pi\frac{n+1}{n}\right) - \sin\left(\pi\frac{n+2}{n+1}\right)\right)$ If it converges, find its value.
   (d) $\sum \frac{n^2-n+2}{\sqrt[4]{n^{10}+n^5+3}}$
   (e) $\sum(-1)^n \frac{1+e^{-n}}{n}$

(f) $\sum n \ln\left(1+\frac{1}{n^3}\right)$

2. Find a power series representation for each of the following.

   (a) $\ln(1-x)$

   (b) $\ln\left(\frac{1+x}{1-x}\right)$

   (c) $\frac{1}{\sqrt{x}}$ centered at $x=1$

3. Find a Maclaurin series for $\frac{1}{\sqrt{4+x}}$. Hint: while you can certainly do this by just taking derivatives and finding the pattern, you can also find a Maclaurin series for $\frac{1}{\sqrt{1+x}}$ by some other means and then modify.

4. Find the radius and interval of convergence for $\sum_{n=2}^{\infty}(-1)^n\frac{(4x+1)^n}{n}$

5. (a) Find a Taylor series for $f(x)=e^x$ centered at 3.

   (b) Determine the radius and interval of convergence of your answer to (a)

   (c) Show that your answer to (a) actually equals e^x for all x

Formula Sheet

Trig Stuff

Product Rules:

$$\sin A\cos B = \tfrac{1}{2}(\sin(A-B)+\sin(A+B))$$
$$\sin A\sin B = \tfrac{1}{2}(\cos(A-B)-\cos(A+B))$$
$$\cos A\cos B = \tfrac{1}{2}(\cos(A-B)+\cos(A+B))$$

Double/Half Angle:

$$\sin x\cos x = \tfrac{1}{2}\sin(2x) \qquad \sin(2x)=2\sin(x)\cos(x)$$
$$\sin^2 x = \tfrac{1}{2}(1-\cos(2x)) \qquad \cos(2x)=1-2\sin^2(x)$$
$$\cos^2 x = \tfrac{1}{2}(1+\cos(2x)) \qquad \cos(2x)=2\cos^2(x)-1$$
$$\tan(2x)=\tfrac{2\tan(x)}{1-\tan^2(x)} \qquad \cos(2x)=\cos^2(x)-\sin^2(x)$$

Other:

$$\int \sec x = \ln(\sec x+\tan x)+C \qquad \int \csc x = \ln(\csc x-\cot x)+C$$
$$\tfrac{d}{dx}\sin^{-1}x = \tfrac{1}{\sqrt{1-x^2}} \qquad \tfrac{d}{dx}\cos^{-1}x = -\tfrac{1}{\sqrt{1-x^2}}$$

Arclength/Surface Area

$$L=\int_a^b \sqrt{1+(f'(x))^2} \qquad A=2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}$$

Variation of Parameters

$$u_1' = -\frac{y_2G(x)}{a(y_1y_2'-y_1'y_2)} \qquad u_2' = \frac{y_1G(x)}{a(y_1y_2'-y_1'y_2)}$$

Taylor Series

$$\sin x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} \qquad \cos x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} \qquad e^x=\sum_{n=0}^{\infty}\frac{x^n}{n!}$$

$$\ln(1+x)=\sum_{n=1}^{\infty}(-1)^{n+1}\frac{x^n}{n} \qquad \tan^{-1}x=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} \qquad (1+x)^k=\sum_{n=0}^{\infty}\frac{k(k-1)\cdots(k-(n-1))}{n!}x^n$$

$$R_n(x)=\frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1} \text{ for some } z \text{ between } a \text{ and } x$$