Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Power Series

1. Find the radius and interval of convergence for each of the following power series:

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n2^n}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(2x+1)^n}{\sqrt[3]{n+2}}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{(x-2)^n}{(n+2)(n-3)}$

   (d) $\displaystyle\sum_{n=1}^{\infty} n!(x-1)^n$

   (e) $\displaystyle\sum_{n=1}^{\infty} \frac{x^n}{n^n}$

2. Show that if $\lim_{n\to\infty} \sqrt[n]{|c_n|}$ converges to a value c, then the radius of convergence of $\sum c_n x^n$ is $\frac{1}{c}$

3. True/False. Justify your answer with a proof or a counterexample.

   (a) If $\sum c_n 2^n$ converges, then $\sum c_n(-2)^n$ converges

   (b) If $\sum c_n(-4)^n$ converges, then $\sum c_n 3^n$ converges

   (c) If $\sum c_n(-4)^n$ diverges, then $\sum c_n 3^n$ diverges

   (d) If $\sum c_n$ converges and $|x| < 1$, then $\sum c_n x^n$ converges

   (e) If $\sum c_n x^n$ has positive radius of convergence, then $\lim c_n = 0$

   (f) Given the interval of convergence of a power series, you can determine the radius of convergence.

   (g) Given the radius of convergence of a power series, you can determine the interval of convergence.

Extra Problems If you finish early, take a stab at these.

1. As you probably know, there are infinitely many prime numbers. Let's prove it:

   (a) Consider the series you would get by multiplying out $(1+\frac{1}{2}+\frac{1}{4}+\cdots)(1+\frac{1}{3}+\frac{1}{9}+\cdots)$. In terms of their prime factorization, what numbers would appear as denominators?

   (b) Do the same for $(1+\frac{1}{2}+\frac{1}{4}+\cdots)(1+\frac{1}{3}+\frac{1}{9}+\cdots)(1+\frac{1}{5}+\frac{1}{25}+\cdots)$

   (c) By extrapolating from (a) and (b), what's another way of writing the product you get from using all the primes? ie, $(1+\frac{1}{2}+\frac{1}{4}+\cdots)(1+\frac{1}{3}+\frac{1}{9}+\cdots)(1+\frac{1}{5}+\frac{1}{25}+\cdots)(1+\frac{1}{7}+\frac{1}{49}+\cdots)(1+\frac{1}{11}+\frac{1}{121}+\cdots)\cdots$ Does this converge or diverge?

   (d) Using the fact that each multiplicand has finite value (what is it?), show that there must be infinitely many prime numbers.

2. (A closed form for the Fibonacci sequence) The Fibonacci sequence is defined by $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$

   (a) Use induction to show that if x satisfies the equation $x^2 = x + 1$, then $x^n = xF_n + F_{n-1}$ for any $n \geq 2$.
   Hint: $x^{n+1} = xx^n$

(b) Let $y = \frac{-1+\sqrt{5}}{2}$, $z = \frac{-1-\sqrt{5}}{2}$ be the two roots of $x^2 = x + 1$. From part (a), we know that $y^n = yF_n + F_{n-1}$ and that $z^n = zF_n + F_{n-1}$. Subtract these equations and plug in the values of y and z to find a closed form for F_n.

(c) Is it even obvious that your closed form evaluates to an integer?