Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Sequences

1. For each of the following sequences, write the form of the general term a_n, starting your indexing at $n = 1$. Also determine whether each sequence is convergent or divergent. For those that are convergent, find the limit.
   (a) $\{1, 2, 3, 4, \ldots\}$
   (b) $\{2, -2, 2, -2, 2, \ldots\}$
   (c) $\{4, 7, 10, 13, \ldots\}$
   (d) $\{\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, -\frac{1}{16}, \ldots\}$
   (e) $\{-\frac{1}{2}, \frac{2}{3}, -\frac{3}{4}, \frac{4}{5}, \ldots\}$
2. Determine whether each of the following sequences are convergent or divergent. For those that are convergent, find the limit.
   (a) $a_n = \frac{3n^2+1}{n^2-1}$
   (b) $a_n = \frac{(n+2)!}{n^2 \cdot n!}$
   (c) $a_n = \cos 2\pi n$
   (d) $\{1, \frac{1}{2}, 1, \frac{1}{4}, 1, \frac{1}{8}, \ldots\}$
   (e) $a_n = \ln(n^2 - 3n + 1) - \ln(n^2 + 4)$
   (f) $a_n = \frac{\sin n}{n}$
   (g) $a_n = n \tan(1/n)$
3. Consider the sequence $a_n = r^n$, where r is a constant.
   (a) Write out a few terms of this sequence for $r = -2, -1, 1/2, 1, 2$. What is the limit in each of these cases?
   (b) In general, for what values r does this sequence converge? Find the limit for those values.
   (c) Repeat part (b) for the sequence $a_n = nr^n$.
4. Let $p(x) = b_l x^l + b_{l-1} x^{l-1} + \cdots + b_0, q(x) = c_m x^m + c_{m-1} x^{m-1} + \cdots + c_0$ be polynomials of degrees l, m respectively. Define a sequence a_n by $a_n = \frac{p(n)}{q(n)}$. Determine whether $\lim_{n\to\infty} a_n$ exists in each of the following cases. When it does, find its value.
   (a) $\deg p < \deg q$
   (b) $\deg p = \deg q$
   (c) $\deg p > \deg q$
5. True/False. For all problems, a_n and b_n are sequences. Justify your answers with a sketch of a proof or a counterexample.
   (a) If a_n and b_n converge, then $a_n + b_n$ converges.
   (b) If $a_n + b_n$ converges, then a_n and b_n converge.
   (c) If a_n and b_n converge, then a_n/b_n converges.
   (d) If a_n and b_n diverge, then $a_n + b_n$ diverges.
   (e) If $a_n + b_n$ diverges, then a_n and b_n diverge.
   (f) If a_n and b_n diverge, then $a_n b_n$ diverges.

Recursively Defined Sequences

1. For each of the following recursively defined sequences, find a closed form for a_n. Use your answer to determine if each is convergent or divergent.

   (a) $a_n = na_{n-1};\ a_1 = 1$

   (b) $a_n = a_{n-1} + n;\ a_1 = 1$

   (c) $a_n = ra_{n-1};\ a_1 = a$ (here a and r are constants)

   (d) $a_n = \ln n + a_{n-1};\ a_1 = 0$

2. (a) Explain why $\lim_{n\to\infty} a_n = \lim_{n\to\infty} a_{n+1}$ for any convergent sequence a_n

   (b) Let a_n be the sequence $\{\sqrt{2}, \sqrt{2\sqrt{2}}, \sqrt{2\sqrt{2\sqrt{2}}}, \dots\}$. Use part (a) to find $\lim_{n\to\infty} a_n$, assuming it exists.