

Math 1B MT2 Practice Problems 2

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Work through the following practice midterm as a group. As always with practice tests, the inclusion or absence of certain types of problems should not be taken as an indication of what will or will not be on the actual midterm. And don't freak out: this is longer than the actual midterm will be.

1. Quick Answers

- (a) Identify each of the following as (L) linear, (S) separable, (B) both, or (N) neither.
 - i. $y' = xy$
 - ii. $xy' = y(y+1)\cos x$
 - iii. $yy' = xy$
 - iv. $yy' = x + y$
- (b) Sketch a few solution curves to $y' = y(y-1)(y+2)$, making sure to clearly indicate all equilibrium solutions.
- (c) Write the form of a particular solution to each of the following.
 - i. $y'' + y = e^x$
 - ii. $y'' + y = \cos x$
 - iii. $y'' + y = e^x \cos x + x^2 e^x \sin x$
 - iv. $y'' + 2y' + y = e^{-x}$ (be careful!)
 - v. $y'' + 2y' + y = \sin x + 5 + x + e^x$

2. Consider the differential equation $y'' + 4y = 0$

- (a) Find the general solution
- (b) For each of the following boundary conditions, determine whether there is one solution, no solutions, or infinitely many solutions to the associated boundary value problem. For one or infinitely many, find the solution
 - i. $y(0) = 0, y(\pi) = 1$
 - ii. $y(0) = 0, y(\pi) = 0$
 - iii. $y(0) = 0, y(\pi/4) = 1$

3. Find the general solution to each of the following

- (a) $y' + 2xy = y$
- (b) $y' - 2xy = 3t^2 e^{t^2}$
- (c) $x^2 y' = x^2 + y^2 + yx$
- (d) $y' + y = xy^3$
- (e) $y'' - 2y' + y = \cos x + 3 \sin x$
- (f) $y'' + y = \csc x$
- (g) $y'' + y = x \sin x$

4. A 100L tank initially contains 10kg of salt. A bag of salt is poured into the tank at a rate of 1kg/min. The solution is kept thoroughly mixed and 2L/min drains from the tank.

- (a) Setup an initial value problem whose solution will give the amount of salt in the tank at time t
- (b) Solve your differential equation from part (a)