

Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Springs and Things

- Recall that the equation for a damped spring is $my'' + cy' + ky = F(t)$, where $F(t)$ is the external force being applied to the system. Find an equation for the displacement of a spring with a mass of $1kg$, a damping coefficient of $4kg/s$ and a spring constant of $25kg/s^2$, driven by an external force $f(t) = \tan t$
- Suppose we use the spring equation above to model the motion of the shock absorbers in a bicycle after hitting a pothole. For simplicity, let's assume it's a really old bike that has very little damping force left (ie, $c=0$). Suppose you try to "ride out" the pothole by gently hopping up and down on your bike right as you hit the bump so the external force you are applying is $F(t) = F_0 \cos \omega_0 t$. Also, let ω be the natural frequency of the shock absorbers.
 - If $\omega \neq \omega_0$, use the method of undetermined coefficients to find an equation for the motion of the shock absorbers.
 - Now suppose that by some great cosmic coincidence you happen to jump up and down at the same rate as your shock absorbers naturally would have (ie, $\omega_0 = \omega$). Now find a new equation for the motion of your bike.
 - What is fundamentally different about the solutions to part (b) versus part (a)?
- Show that if a, b, c are all greater than 0, then all solutions to $ay'' + by' + cy = 0$ have the property that $\lim_{x \rightarrow \infty} y(x) = 0$.
Hint (for all three parts): break this down into cases based on the discriminant.
 - If $a > 0, c > 0$ but $b = 0$, show that the result from part (a) is no longer true, but that all solutions are bounded as $x \rightarrow \infty$. Bounded means that the solutions have a limited range of values.
 - If $a > 0, b > 0$ and $c = 0$, show that all solutions approach some constant (not necessarily 0) as $x \rightarrow \infty$. Determine this constant in terms of $y(0)$, which we'll call a , and $y'(0)$, which we'll call b
- (Blatantly stolen from Theo) In addition to springs, we can also use 2nd order ODEs to model the flow of current through a circuit consisting of a resistor of R ohms, a capacitor of C farads, and an inductor of L Henrys. In particular, if $Q(t)$ represents the charge in the circuit at time t , then Q satisfies the differential equation $LQ'' + RQ' + \frac{1}{C}Q = E(t)$, where $E(t)$ is the voltage being applied to the system.
 - Suppose that $R^2 < 4L/C$ and that $E(t) = 0$. Sketch a graph of Q and describe how Q changes over time.
 - Now suppose that instead of $E(t) = 0$, AC current obeying the equation $E(t) = \sin \omega t$ is applied to the circuit. Find $\lim_{t \rightarrow \infty} Q(t)$.
 - Suppose $R = 0$ and $E(t) = \sin \omega t$. For what value of ω will $Q(t)$ be unbounded (ie, what ω will cause Q to get bigger and bigger as time goes on)?

Extra Problems: If you finish early, take a stab at some of these slightly more challenging/interesting problems:

- When taking derivative and integrals of functions involving complex numbers, we just do the same thing as for real-valued functions, treating complex constants the same as real valued constants.
 - Find $\frac{d}{dx} e^{(a+bi)x}$
 - Remembering that integrals just signify anti-derivatives, find $\int e^{(1+i)x} dx$. By properly calculating $\frac{1}{1+i}$, re-write your answer in the form $f(x) + ig(x)$
 - By equating real and imaginary parts and using Euler's formula, use your answer to (b) to find $\int e^x \cos x dx$ and $\int e^x \sin x dx$