

# Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

## Variation of Parameters

1. Use variation of parameters to find the general solution to  $y'' - y = \frac{1}{x}$
2. Use variation of parameters to find the general solution to  $y'' + y = \sec^2 x$ . You may want to write down your answer in order to help with number 3.
3. Use a combination of variation of parameters and undetermined coefficients to find the general solution to  $y'' + y = e^x + \sec^2 x$

## Leftover Undetermined Coefficients Problems

4. Consider the equation  $y'' + y = xe^x + \cos x$ . Note that you will have 4 undetermined coefficients if you try to solve this directly. In order to avoid this mess, it's sometimes easier to solve equations like this separately and then add the results. Find the general solution to this equation by first finding particular solutions to  $y'' + y = xe^x$  and  $y'' + y = \cos x$  and then adding them.
5. When doing undetermined coefficients, sometimes even multiplying your guess by  $x$  will not be enough, but luckily you can always just multiply by  $x$  and try again. Use this technique to find the general solution to  $y'' - 8y' + 16y = e^{4x}$

**Extra Problems:** If you finish early, take a stab at some of these slightly more challenging/interesting problems:

1. (a) Show that if  $a, b, c$  are all greater than 0, then all solutions to  $ay'' + by' + cy = 0$  have the property that  $\lim_{x \rightarrow \infty} y(x) = 0$   
(b) If  $a > 0, c > 0$  but  $b = 0$ , show that the result from part (a) is no longer true, but that all solutions are bounded as  $x \rightarrow \infty$ . Bounded means that the solutions have a limited range of values.  
(c) If  $a > 0, b > 0$  and  $c = 0$ , show that all solutions approach some constant (not necessarily 0) as  $x \rightarrow \infty$ . Determine this constant in terms of  $y(0)$ , which we'll call  $a$ , and  $y'(0)$ , which we'll call  $b$
2. When taking derivative and integrals of functions involving complex numbers, we just do the same thing as for real-valued functions, treating complex constants the same as real valued constants.
  - (a) Find  $\frac{d}{dx} e^{(a+bi)x}$
  - (b) Remembering that integrals just signify anti-derivatives, find  $\int e^{(1+i)x} dx$ . By properly calculating  $\frac{1}{1+i}$ , re-write your answer in the form  $f(x) + ig(x)$
  - (c) By equating real and imaginary parts and using Euler's formula, use your answer to (b) to find  $\int e^x \cos x dx$  and  $\int e^x \sin x dx$