Math 1B Discussion Section Problems

Rob Bayer

July 17, 2008

You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Undetermined Coefficients

1. For each of the following possibilities for $G(x)$, decide if undetermined coefficients would work. For those where it would, write the form of your guess:
   (a) e^{-3x}
   (b) e^{x^2}
   (c) $\sin 3x$
   (d) x
   (e) $\cos x - \sin x$
   (f) $\frac{\sin x}{x}$
   (g) $x^2 e^{-7x} \sin 6x$
   (h) $\frac{e^x}{x^2+1}$
   (i) $x + 3$
   (j) $\ln x$
2. Find the general solution to each of the following:
   (a) $y'' + 2y' - 3y = e^{2x}$
   (b) $y'' + 2y' + y = e^x \cos x$
   (c) $y'' + 2y' + y = xe^x$
3. Solve the initial value problem $y'' + y' - 2y = e^x;\ \ y(0) = 0, y'(0) = 1$

   Note: you probably won't be able to do problems after this point until the second half of class.

4. Write the form of your guess for y_p when solving $y'' - y = g(x)$ where $g(x)$ is each of the following. Make sure to multiply by x when necessary:
   (a) $x^2 e^{-7x} \sin 6x + 7 - x \cos 4x$.
   (b) $x + 3 + e^x$
   (c) $(x + 2)e^x$
5. Consider the equation $y'' - 5y' + 6 = xe^x + \cos x$. Note that you will have 4 undetermined coefficients if you try to solve this directly. In order to avoid this mess, it's sometimes easier to solve equations like this separately and then add the results. Find the general solution to this equation by first finding particular solutions to $y'' - 5y' + 6 = xe^x$ and $y'' - 5y' + 6 = \cos x$ and then adding them.
6. Solve the initial value problem $y'' + y' - 2y = e^{2x};\ \ y(0) = 0, y'(0) = 1$
7. Sometimes, even multiplying your guess by x will not be enough, but luckily you can always just multiply by x and try again. Use this technique to find the general solution to $y'' - 8y' + 16y = e^{4x}$

Extra Problems: If you finish early, take a stab at some of these slightly more challenging/interesting problems:

1. (a) Show that if a, b, c are all greater than 0, then all solutions to $ay'' + by' + cy = 0$ have the property that $\lim_{x\to\infty} y(x) = 0$

(b) If $a > 0, c > 0$ but $b = 0$, show that the result from part (a) is no longer true, but that all solutions are bounded as $x \to \infty$. Bounded means that the solutions have a limited range of values.

(c) If $a > 0, b > 0$ and $c = 0$, show that all solutions approach some constant (not necessarily 0) as $x \to \infty$. Determine this constant in terms of $y(0)$, which we'll call a, and $y'(0)$, which we'll call b

2. When taking derivative and integrals of functions involving complex numbers, we just do the same thing as for real-valued functions, treating complex constants the same as real valued constants.

(a) Find $\frac{d}{dx}e^{(a+bi)x}$

(b) Remembering that integrals just signify anti-derivatives, find $\int e^{(1+i)x}dx$. By properly calculating $\frac{1}{1+i}$, re-write your answer in the form $f(x) + ig(x)$

(c) By equating real and imaginary parts and using Euler's formula, use your answer to (b) to find $\int e^x \cos x dx$ and $\int e^x \sin x dx$