Math 1B Discussion Section Problems

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July 16, 2008

You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Complex Numbers

1. Consider the equation $x^2 + 6x + 13 = 0$
   (a) Solve it. Call the two solutions r and s.
   (b) Compute s^2. Write your answer in the form $a + bi$
   (c) Find s^{-2}. Again, write your answer in the form $a + bi$
   (d) Plot $s^2, 6s$,and 13 as vectors in the Complex plane. Show geometrically that these vectors add to 0.

2. Suppose $z = x + iy$ is a complex number.
   (a) Draw $x + iy$ as some point in the complex plane. (It's probably easiest if you put it somewhere in the first quadrant)
   (b) Find the distance between z and the origin.
   (c) Find the angle between z and the x-axis. Hint: draw a right triangle
   (d) How can you use parts (b) and (c) to write z as $re^{i\theta}$?

3. Show that if $z = r(\cos\theta + i\sin\theta)$, then $z^n = r^n(\cos(n\theta) + i\sin(n\theta))$. Hint: use Euler's formula to write z in a more convenient form

4. Use Euler's Formula to show that

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \qquad \sin x = \frac{e^{ix} - e^{-ix}}{2i}$$

5. When taking derivative and integrals of functions involving complex numbers, we just do the same thing as for real-valued functions, treating complex constants the same as real valued constants.
   (a) Find $\frac{d}{dx}e^{(a+bi)x}$
   (b) Remembering that integrals just signify anti-derivatives, find $\int e^{(1+i)x}dx$. By properly calculating $\frac{1}{1+i}$, re-write your answer in the form $f(x) + ig(x)$
   (c) By equating real and imaginary parts and using Euler's formula, use your answer to (b) to find $\int e^x \cos x dx$ and $\int e^x \sin x dx$

Complex Roots

1. Find the general solution to $y'' - 6y' = -9y$

2. Solve the initial value problem $y'' - 6y' + 13y = 0$ where $y(0) = 4$ and $y'(0) = 0$

3. Solve the boundary value problem $y'' + 4y = 0$; $y(0) = 0, y(\pi/4) = 1$? What would happen if it had been $y(\pi) = 1$ instead? Moral of the story: **all** initial value problems have solutions, not all boundary value problems do.