Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Linear Differential Equations

1. (a) Show that $xy' = y$ is separable, of homogeneous type, and linear by putting it into the appropriate form for each method.
   (b) Solve the equation using each of the three methods. Do you get the same thing each way?
2. For each of the following equations, decide whether it is separable, linear, of homogeneous type, some combination of the 3, or none. For those that are linear, find the general solution
   (a) $yy' = x\sqrt{1+x^2}\sqrt{1+y^2}$
   (b) $xy' - 2y = x^3$
   (c) $xy' = y + x\cos^2(y/x)$
   (d) $1 + y^2 - y'\sqrt{1-x^2} = 0$
   (e) $y' = x + y$
   (f) $1 + 2xy^2 + 2x^2yy' = 0$
3. Consider the differential equation $y' = 3x(y + x^n)$.
   (a) For what values of n is this equation separable?
   (b) Linear?
4. Imagine a large tank that starts with 1000L of pure water, has 5L/min of 1kg/L saltwater entering it, and has 6L/min of solution leaving through a drain in the bottom.
   (a) What makes this problem different than the ones we did before?
   (b) Find a differential equation whose solution will give us the amount of salt (in kg) in the tank at time t.
   (c) What's different about this differential equation from the examples we saw before?
   (d) Solve your differential equation using whatever method seems best to you.
   (e) How much salt is there when there is only 1L of water left in the tank?
5. A circuit containing an inductor of inductance L and a resistor of resistance R obeys the differential equation $LI' + RI = V(t)$, where $I(t)$ is the current at time t and $V(t)$ is the voltage driving the circuit. The voltage in a standard wall outlet can be modeled as $V(t) = 120\sin(120\pi t)$, where t is measured in seconds. Solve this differential equation to find the current flowing through the circuit at time t given that $I(0) = 0$.

Bernoulli

1. Show that the substitution $u(x) = y^{1-n}$ will **always** transform $y' + P(x)y = Q(x)y^n$ into a linear equation. It may be helpful to start by dividing through by y^n.
2. Use this method to solve the differential equation $y' + xy = xe^{-x^2}y^{-3}$

Extra/Hard Problems If you finish early, take a stab at these.

1. Consider the differential equation $y' + xy = x^3$.

   (a) What is the general solution to this equation?

   (b) Let y_1 and y_2 be any two (distinct) solutions to this differential equation, and let $y_0 = y_1 - y_2$. Is y_0 a solution to this ODE? If yes, prove it. If not, what differential equation does it satisfy?

   (c) Now do the same for $y_0 = cy_1$, where c is any constant.

2. (Make sure you've done 1 first)

   (a) Prove that if y_1 and y_2 are solutions to $y' + p(x)y = 0$, then $y_0 = cy_1 + y_2$ is also a solution to the same equation for any choice of the constant c.

   (b) Show that $1/x$ and 0 both satisfy the differential equation $y' + y^2 = 0$, but that $c/x + 0$ does not unless $c = 0$ or $c = 1$. Why does this not contradict part (a)?

3. Show that if r is a double root of $p(x)$, then r is also a root of $p'(x)$. Hint: r is a double root if and only if $p(x)$ is divisible by $(x - r)^2$