Math 1B Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Separable word problems

1. A certain curve in the plane has the property that **every** normal line (that is, a line perpendicular to the tangent line) to the curve passes through (2,0). Find the equation for this curve if you know it passes through (1,1). Hint: recall that two lines are perpendicular if and only if the product of their slopes is -1.

2. (Adapted from Theo Johnson-Freyd and Stewart) Without accounting for fishing, the Pacific Halibut Fishery can be modeled by the differential equation $y' = ky(M - y)$, where $y(t)$ is the biomass at time t, $M = 8 \times 10^7 kg$ is the carrying capacity, and $k = 8.875 \times 10^{-9}$.

   (a) Suppose that in addition to the natural birth and death of fish, fishing companies also harvest H fish per year. Come up with a new differential equation that models this situation and determine its equilibria.

   (b) As a general rule, we want fishery populations to remain constant, so the managing agencies allow fisherman to harvest the same number of fish each year as the population would naturally grow on its own. What should H be? Solve this differential equation under these conditions.

   (c) Assuming we want to provide the maximum possible profits for fishing companies by allowing as much fishing as possible, what is the optimal population of Halibut?

   (d) Using similar reasoning as in part (c), at what population does a species that obeys the logistic equation grow most rapidly?

3. Last time we saw that one model for the fall of a baseball is $v' = g - bv^2$. Solve this equation for v and then use your answer to find an equation for the height of the ball at time t.

4. The differential equation $\dfrac{dy}{dx} = ky^{1+a}$ is sometimes called the Doomsday Equation. Here we'll try to figure out why:

   (a) Solve this equation in terms of k, a, and an arbitrary constant C. How is this different than the solution to $y' = ky$?

   (b) Now suppose the growth of a population of rabbits follows the differential equation $y' = 2y^{1.01}$ and suppose there are initially only two rabbits. When should we all start running for the hills?

First Order ODEs of Homogeneous Type

1. Find the general solution to $x^2y' = y^2 + 3yx + x^2$

2. Solve the initial value problem $y' = \tan(y/x) + \dfrac{y}{x}$; $y(1) = \pi/2$