Math 1B Discussion Section Problems

Rob Bayer

July 8, 2008

You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Separable Equations

1. Solve each of the following separable equations/initial value problems:
   (a) $y' = xe^y$
   (b) $\dfrac{dy}{dx} = e^{x+y}(e^x + 1)^{-1}$
   (c) $y' = \dfrac{y \cos x}{1 + y^2}$; $y(0) = 1$
   (d) $y' = \dfrac{e^y \sin^2 \theta}{y \sec \theta}$
   (e) $y' = e^x + e^x y^2$
   (f) $y' = 2 + 2y + t + ty$; $y(0) = -1$
2. Let $y(t)$ satisfy the initial value problem $y' = y(y-2)(y+3)$; $y(0) = 1$. What is $\lim_{t\to\infty} y(t)$? What if $y(0) = 3$? Hint: while you could solve this differential equation explicitly, there is a **much** easier way to solve this problem.
3. Like most things, it turns out that the temperature function $T(t)$ of a Turkey in an oven obeys the differential equation $\frac{dT}{dt} = k(T_{oven} - T)$, where k is some arbitrary constant and T_{oven} is the (constant) temperature of the oven.
   (a) Solve the differential equation to get an explicit formula for $T(t)$ in terms of k, T_{oven} and some arbitrary constant C
   (b) Suppose you put a $20°C$ Turkey in a $205°$ oven and after 30 minutes it has warmed to $32°$. Use the given initial condition and the temperature after 30 minutes to find actual values for k and C
   (c) How long does it take before the turkey reaches a well-done $82°$?

Separable word problems

1. A certain curve in the plane has the property that **every** normal line (that is, a line perpendicular to the tangent line) to the curve passes through (2,0). Find the equation for this curve if you know it passes through (1,1). Hint: recall that two lines are perpendicular if and only if the product of their slopes is -1.
2. (Adapted from Theo Johnson-Freyd and Stewart) Without accounting for fishing, the Pacific Halibut Fishery can be modeled by the differential equation $y' = ky(M - y)$, where $y(t)$ is the biomass at time t, $M = 8 \times 10^7 kg$ is the carrying capacity, and $k = 8.875 \times 10^{-9}$.
   (a) Suppose that in addition to the natural birth and death of fish, the fishery also allows the fishing companies to harvest H fish per year. Come up with a new differential equation that models this situation and determine its equilibria.
   (b) As a general rule, the fishery wants their population to remain constant, so they allow fisherman to harvest the same number of fish each year as the population would naturally grow on its own. What should H be? Solve this differential equation under these conditions.

(c) Assuming the fishery wants to maximize its profits by allowing as much fishing as possible, what is the optimal population of Halibut? Note: there are multiple ways to do this problem, the most straightforward of which involves looking at y''. Without solving the actual differential equation, how can you find y''?

(d) Using similar reasoning as in part (c), at what population does a species that obeys the logistic equation grow most rapidly?

3. Last time we saw that one model for the fall of a baseball is $v' = g - bv^2$. Solve this equation for v and then use your answer to find an equation for the height of the ball at time t.

4. The differential equation $\frac{dy}{dx} = ky^{1+a}$ is sometimes called the Doomsday Equation. Here we'll try to figure out why:

   (a) Solve this equation in terms of k, a, and an arbitrary constant C. How is this different than the solution to $y' = ky$?

   (b) Now suppose the growth of a population of rabbits follows the differential equation $y' = 2y^{1.01}$ and suppose there are initially only two rabbits. When should we all start running for the hills?