

Math 1B Discussion Section Problems

Rob Bayer

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Intro to Differential Equations

1. For each differential equation, determine if the given function is a solution:

(a) $y' = e^x + y$; $y = xe^x$

(b) $\frac{dP}{dx} = 1 + P^2$; $P = \tan x$

(c) $(y')^2 = 4 + y^2$; $y = e^x - e^{-x}$

(d) $f' = \frac{1}{e^f}$; $f = \ln(x + C)$

2. The air in a room with volume 200 m^3 contains .15% CO_2 . After opening a window, fresh air with only 0.05% CO_2 enters the room at a rate of $2 \text{ m}^3/\text{min}$ and mixed air flows out of the room at the same rate. Set up an initial value problem for the amount of CO_2 in the room.

3. (a) Find the general solution for each of the following differential equations:

i. $\frac{dy}{dx} = e^x$

ii. $y'' = 20x^3 + 2$

(b) Solve the initial value problem $y' = e^x$, $y(0) = 3$

(c) How many initial conditions would you need in order to solve an initial value problem for part (ii)?

4. Show that $y = C \sec x + C \tan x$ is a 1 parameter family of solutions to $y = y \sec x$. If $y(\pi/4) = 5$, what is C ?

Direction Fields/Qualitative Analysis

1. Do number 5 first

2. Suppose a function $y(t)$ satisfies the differential equation $y' = y^3 - 6y^2 + 5y$.

(a) What are the constant/equilibrium solutions?

(b) When is y increasing?

(c) Decreasing?

(d) Using your information from (a), (b), and (c) try to vaguely sketch a few solutions to this differential equation.

3. Let's try to figure out what would happen if we were to drop a baseball off a really tall building.

Some physics: From Newton's Second Law, we know that the $F = ma = mv'$, where F is the total force acting on the baseball, m is its mass, and $v = v(t)$ is its velocity. We also know that the total force acting on the baseball is $F = F_{gravity} - F_{resistance} = mg - bv^2$, where g is the acceleration due to gravity and b is the friction coefficient.

(a) Explain why the above information suggests that $v' = g - \frac{b}{m}v^2$ would be a good differential equation to model the fall of a baseball.

(b) Find the equilibrium solutions to this equation and sketch a few solution curves. Are your equilibrium stable or unstable?

(c) What happens to your equilibrium if you increase or decrease the mass of the baseball? Does this make sense intuitively?

4. Consider the differential equation $y' = \frac{x}{y}$.
- (a) In what regions of the xy -plane is y' positive? Negative? Where is it 0?
 - (b) Sketch the direction field for this equation.
 - (c) Find all solutions of the form $y = mx + b$. Note that simply looking at your picture is not really a proof—you need to do actual math to find them!
5. Consider the following direction fields:
- (a) Without thinking hardly at all, which one of these is for $y' = 1 + y$? How can you tell?
 - (b) The differential equations for the other ones are $y' = x^2 - y^2$, $y' = y \sin(2x)$, and $y' = 1 - xy$. Determine which is which.
 - (c) Using the direction fields, sketch some solution curves to $y' = x^2 - y^2$.