Math 1B Group Work Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Surface area

1. A leaky 10-kg bucket is lifted from the ground to a height of 12m at a constant speed with a rope that weighs 0.8 kg/m. Initially the bucket contains 36kg of water, but the water leaks at a constant rate and finishes draining just as the bucket reaches the 12m level. How much work is done?

2. Newton's Law of Gravitation states that the force of gravity between two objects of masses m_1 and m_2 a distance r apart is $F = 6.67 \times 10^{-11} \frac{m_1 m_2}{r^2}$. Find the work required to launch a 1000kg satellite to an orbit 1000km above the Earth's surface. The mass of the earth is approximately 5.98×10^{24} kg and its radius is approximately 6.37×10^6 m.

3. Find the center of mass of the region enclosed by $y = 4 - x^2$ and $y = 0$

Integration Practice

1. Integrate each of the following:
   (a) $\int e^{x+e^x} dx$
   (b) $\int \frac{\sec^2(\sin\theta)}{\sec\theta} d\theta$
   (c) $\int \frac{1}{\sqrt{x+1}+\sqrt{x}} dx$
   (d) $\int \frac{\ln(x+1)}{x^2} dx$
   (e) $\int \frac{t^3+1}{t^3-t^2} dt$
   (f) $\int \cos^3 2x \sin 2x dx$
   (g) $\int \frac{dt}{\sqrt{e^t}}$
   (h) $\int \frac{1}{x\sqrt{x^2+4}} dx$
   (i) $\int \frac{1}{x\sqrt{x+4}} dx$
   (j) $\int \frac{x}{\sqrt{x^2+4}} dx$
   (k) $\int \frac{1}{\sqrt[4]{x}+\sqrt[3]{x}}$
   (l) $\int \sqrt{\frac{x-1}{x+1}} dx$

2. It turns out that **any** rational function of sin and cos can be integrated by using the "Weierstrass substitution," $t = \tan(\frac{x}{2})$
   (a) Show that $\cos\frac{x}{2} = \frac{1}{\sqrt{1+t^2}}, \sin\frac{x}{2} = \frac{t}{\sqrt{1+t^2}}$.
   (b) Use the half-angle formulas to show that $\sin x = \frac{2t}{1+t^2}, \cos x = \frac{1-t^2}{1+t^2}$
   (c) Use the Weierstrass substitution to find $\int \frac{1}{3\cos x + 4\sin x} dx$
   (d) Convince yourselves that this method will let you integrate any rational function of sine and cos.