

Math 1B Group Work Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Surface area

1. Find the surface area of the solid obtained by rotating $y = \ln(x^2 + 1)$ about the x-axis for $0 \leq x \leq 1$.
2. Consider the function $f(x) = 1/x$ for $1 \leq x < \infty$
 - (a) Show that the area under this curve is infinite
 - (b) Show that the length of this curve is infinite
 - (c) If you rotate this curve around the x-axis, you get a solid known as Gabriel's Horn or Torcelli's Trumpet. Sketch a picture of the trumpet and show that its surface area is infinite.
 - (d) Show that despite parts (a) - (c), the **volume** of Gabriel's Horn is actually finite.
 - (e) What does this mean in terms of being able to fill it with paint vs being able to paint the inside?
3. A sphere can be thought of as a half-circle rotated around the x-axis. In the same way, rotating a half-ellipse gives something called an ellipsoid. Set up, but do not attempt to evaluate, the integral for the surface area of the ellipsoid you get by rotating the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ around the x-axis.

Integration Practice

1. $\int e^{x+e^x} dx$
2. $\int \frac{1}{\sqrt{x+1}+\sqrt{x}} dx$
3. $\int \frac{\ln(x+1)}{x^2} dx$
4. $\int \frac{t^3+1}{t^3-t^2} dt$
5. $\int \frac{1}{x\sqrt{x^2+4}} dx$
6. $\int \frac{1}{x\sqrt{x+4}} dx$
7. $\int \frac{x}{\sqrt{x^2+4}} dx$

Extra Problems (if you finish early, take a stab at these)

1. For which values of p does $\int_0^\infty e^{px}$ converge?
2. $\int \frac{1}{x\sqrt{x-1}}$