

Math 1B Group Work Problems

Rob Bayer

June 30, 2008

You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

The Comparison Test

1. Decide what relationship (\leq , \geq , $=$, etc), if any, holds between each of the following pairs of functions on the given intervals:

(a) $x, x^2; 0 \leq x \leq 1$

(b) $\frac{1}{x^2+x}, \frac{1}{x^2}; 1 \leq x < \infty$

(c) $\frac{1}{2x}, \frac{1}{x^2+x}; 0 < x < 1$

(d) $\frac{1}{x \sin x}, \frac{1}{x}; 0 < x < \pi/2$

2. Without actually evaluating the given integrals, determine whether each of the following converges.

(a) $\int_1^\infty \frac{\cos^2 x}{x^2+3}$

(b) $\int_1^\infty \frac{1}{x+e^{2x}} dx$

(c) $\int_3^\infty \frac{1}{x^2-2x^3+x^4} dx$

(d) $\int_0^1 \frac{x^3-2x^2+3\sqrt{x}}{\sqrt[3]{x^{10}-x^7+4x^3}} dx$

Arc Length

1. What is the length of the curve $y = \frac{x^2+1}{2}$ between $x = 0$ and $x = 2$?
2. Find the length of the curve $y = \ln(\cos x)$ for $0 \leq x \leq \pi/3$
3. Is the length of the curve $y = \ln x$ finite on the interval $[0, 1]$?
4. (a) Sketch the graph of the curve $f(x) = x \sin(1/x)$ for $0 < x \leq 1$
(b) Is the arc length of this graph finite or infinite?

Extra Problems (if you finish early, take a stab at these)

1. Determine each of the following, or show that they are not defined:

(a) $\lim_{t \rightarrow \infty} \int_{-t}^t 2x dx$

(b) $\lim_{t \rightarrow \infty} \int_{-t}^{t+1} 2x dx$

(c) $\lim_{t \rightarrow \infty} \int_{-t}^{\sqrt{t^2+1}} 2x dx$

(d) $\int_{-\infty}^{\infty} 2x dx$

2. For which values of p does $\int_0^\infty e^{px}$ converge?

3. $\int \frac{dx}{\sqrt{x-1}}$

4. $\int \frac{1}{x\sqrt{x-1}}$