

Math 1B Group Work Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Improper Integrals

1. Which of the following integrals are improper? For those that are improper, show how you would break them up into the limit of proper integrals. Evaluate the ones with a *.

(a) $\int_0^{\pi/2} \sec x dx$

(b) $\int_{-3}^3 \frac{1}{x^2+1} dx$

(c) * $\int_4^{\infty} e^{-y/2} dy$

(d) $\int_{-\infty}^{\infty} \frac{1}{x^3+1} dx$

(e) * $\int_0^1 \frac{dy}{4y-1}$

(f) * $\int_{-1}^1 \frac{e^x}{e^x-1} dx$

(g) * $\int_0^{\infty} \sin t dt$

2. Consider the integral $\int_{-1}^1 \frac{1}{x^{2/3}}$.

(a) Explain why this integral is improper and evaluate it.

(b) What would you get if you “forgot” that it was improper and just evaluated its anti-derivative at the endpoints? Why does this answer not make any sense?

The Comparison Test

1. Decide what relationship (\leq , \geq , $=$, etc), if any, holds between each of the following pairs of functions on the given intervals:

(a) $x, x^2; 0 \leq x \leq 1$

(b) $\frac{1}{x^2+x}, \frac{1}{x^2}; 1 \leq x < \infty$

(c) $\frac{1}{2x}, \frac{1}{x^2+x}; 0 < x < 1$

(d) $\frac{1}{x \sin x}, \frac{1}{x}; 0 < x < \pi/2$

2. Without actually evaluating the given integrals, determine whether each of the following converges.

(a) $\int_1^{\infty} \frac{1}{x+e^{2x}} dx$

(b) $\int_3^{\infty} \frac{1}{x^2-2x^3+x^4} dx$

(c) $\int_0^1 \frac{x^3-2x^2+3\sqrt{x}}{\sqrt[3]{x^{10}-x^7+4x^3}} dx$

Extra Problems (if you finish early, take a stab at these)

1. For which values of p does $\int_e^{\infty} \frac{dx}{x(\ln x)^p}$ converge?

2. $\int \frac{dx}{\sqrt{x-1}}$

3. $\int \frac{1}{4 \cos x - 3 \sin x}$