

Math 1B Group Work Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. Instead, you should make sure everyone in your group knows **how** to solve all the problems, and not just the answers.

Approximate Integration

- Compute $\int_0^1 \frac{1}{1+x^2} dx$ directly
 - Now approximate the integral using the trapezoid rule with $n = 4$
 - What's the error of this approximation?
 - What error bound does the Trapezoid rule error bound give you?
- For each of the following rules, draw a continuous (but not necessarily differentiable) function whose integral from 0 to 4 would be best approximated by that rule when using $n = 4$. You should pick your functions so that using any of the other rules would give a very bad estimate for the integral
 - Left Endpoint rule
 - Right Endpoint rule
 - Midpoint rule
 - Trapezoid rule
- Show that Simpson's rule has 0 error when used to approximate the area under any polynomial of degree ≤ 3

Extra Problems (if you finish early, take a stab at these)

- Find $\int x\sqrt{x^4 + 1} dx$
- Prove the formula $A = \frac{1}{2}r^2\theta$ for the area of a sector of circle with radius r of θ radians. Hint: the area is the sum of the area of the triangle POQ and the the region PQR. By using trig, you can find the lengths of the line segments OQ and QP.
- (hard) Let $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ be any three distinct points with $x_1 < x_2 < x_3$. Show that if $p(x)$ and $q(x)$ are any polynomials of degree ≤ 3 such that both p and q pass through all three points, then $\int_{x_1}^{x_3} p(x) dx = \int_{x_1}^{x_3} q(x) dx$