Name: ______________________________

Math 1B Quiz 9 SOLUTIONS

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You have 20 minutes to complete this quiz. You must show your work.

Determine whether each of the following series are **absolutely convergent, conditionally convergent, or divergent**. Be sure to clearly **state which tests** you are using.

1. (4 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n n}{n^2+1}$

   $\frac{n}{n^2+1}$ is decreasing and has a limit of 0, so by the Alternating Series Test, this series converges.

   $\lim \frac{\frac{n}{n^2+1}}{1/n} = \lim \frac{n^2}{n^2+1} = 1$ and $\sum \frac{1}{n}$ diverges, so by the limit comparison test, $\sum \left|\frac{n(-1)^n}{n^2+1}\right|$ diverges.

   Thus, the series is conditionally convergent.

2. (3 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n + \cos n^2}{n^3}$

   $\left|\frac{\sin n + \cos n^2}{n^3}\right| \leq \frac{2}{n^3}$ and $\sum \frac{2}{n^3}$ is a convergent p-series, so by the comparison test, $\sum \left|\frac{\sin n + \cos n^2}{n^3}\right|$ converges and thus $\displaystyle\sum_{n=1}^{\infty} \frac{\sin n + \cos n^2}{n^3}$ is absolutely convergent.

3. (3 pts) $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{n^2 e^n}$

   $\lim \left|\frac{a_{n+1}}{a_n}\right| = \lim \left|\frac{(n+1)!}{(n+1)^2 e^{n+1}} \frac{n^2 e^n}{n!}\right| = \lim \left|\frac{(n+1)n^2}{(n+1)^2 e}\right| = \infty$

   so by the ratio test, this series diverges.