Name: ______________________________

Math 1B Quiz 8 SOLUTIONS

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You have 20 minutes to complete this quiz. You must show your work.

1. (3 pts) Determine whether $\sum_{n=2}^{\infty} \frac{n}{e^{n^2}}$ converges or diverges. You may not use any tests we haven't learned yet (ie, don't use the ratio test).

   Let $f(x) = xe^{-x^2}$, so f is positive and decreasing so we can apply the integral test. We will make the substitution $u = -x^2$, so $du = -2xdx$ and we get:

$$\int_2^{\infty} xe^{-x^2}dx = \lim_{t\to\infty} \int_2^t xe^{-x^2}dx = -\frac{1}{2}\lim \int_{-4}^{-t^2} e^u du = -\frac{1}{2}\lim(e^{-t^2} - e^{-4}) = \frac{1}{2e^4}$$

   So by the integral test, $\sum ne^{-n^2}$ converges.

2. (3 pts) Find $\sum_{n=0}^{\infty} \arctan(n+2) - \arctan(n)$, or show it diverges.

$$\begin{aligned} s_n &= \arctan 2 - \arctan 0 + \arctan 3 - \arctan 1 + \arctan 4 - \arctan 2 + \arctan 5 - \arctan 3 \\ &\quad + \cdots + \arctan(n+1) - \arctan(n-1) + \arctan(n+2) - \arctan n \\ &= -\arctan 0 - \arctan 1 + \arctan(n+1) + \arctan(n+2) \end{aligned}$$

   So $\sum_{n=2}^{\infty} \frac{n}{e^{n^2}} = \lim_{n\to\infty} s_n = 0 - \frac{\pi}{4} + \frac{\pi}{2} + \frac{\pi}{2} = \frac{3\pi}{4}$

3. (4 pts) For each of the following, give an example of a pair of series with the given properties:

   Note: there are lots of possibilites for each. Here's what I would have done:

   (a) $\sum a_n$ and $\sum b_n$ converge, $a_n \geq 0, b_n > 0$, and $\sum \frac{a_n}{b_n}$ diverges.
   $a_n = b_n = \frac{1}{n^2}$

   (b) $\sum a_n$ and $\sum b_n$ diverge, but $\sum(a_n + b_n)$ converges.
   $a_n = 1, b_n = -1$

   (c) $\sum a_n$ converges, $\sum b_n$ diverges, $a_n \geq 0, b_n \geq 0$, and $\sum a_n b_n$ diverges.
   $a_n = \frac{1}{n^2}, b_n = n$

   (d) $\sum a_n$ converges, $\sum b_n$ diverges, $a_n \geq 0, b_n \geq 0$, and $\sum a_n b_n$ converges.
   $a_n = 0, b_n = 1$