Name: ______________________________

Math 1B Quiz 7 SOLUTIONS

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You have 20 minutes to complete this quiz. You must show your work.

1. (3 pts) Determine whether the sequence $a_n = \dfrac{2+\sin(n)}{n^2+\ln n}$ converges or diverges. If it converges, find its value. Be sure to clearly state any theorems you use.

   Since $-1 \le \sin n \le 1$, we have $\frac{1}{n^2+\ln n} \le \frac{2+\sin n}{n^2+\ln n} \le \frac{3}{n^2+\ln n}$. Both the left and right hand side of this inequality go to 0 as $n \to \infty$, so by the squeeze theorem, the original sequence must also.

2. (2 pts) Find $\displaystyle\sum_{n=0}^{\infty} \frac{4^{n+1}}{3^{2n}}$, or show it diverges.

$$\sum_{n=0}^{\infty} \frac{4^{n+1}}{3^{2n}} = \sum_{n=0}^{\infty} 4\frac{4^n}{9^n} = \frac{4}{1-\frac{4}{9}} = \frac{36}{5}$$

   Where we note that $|r| = \frac{4}{9} < 1$ so it converges and we can use the geometric series formula.

3. (a) (3 pts) Prove that the sequence defined by $a_n = \sqrt{3a_{n-1}}, a_1 = 1$ converges.

   Bounded We claim that $0 \le a_n \le 3$:

   - BC: $a_1 = 1$ and $0 \le 1 \le 3$
   - IH: $0 \le a_n \le 3$
   - IS: $a_{n+1} = \sqrt{3a_n} \overset{IH}{<} \sqrt{3 \cdot 3} = 3$ and also $a_{n+1} = \sqrt{3a_n} \overset{IH}{>} \sqrt{3 \cdot 0} = 0$

   Monotone $a_{n+1} = \sqrt{3a_n} > \sqrt{a_n a_n} = a_n$

   Thus, $\{a_n\}$ is bounded and monotonic and thus converges.

   (b) (2 pts) Find $\lim\limits_{n\to\infty} a_n$, where a_n is the sequence from part (a).

   Let $L = \lim a_n$. Then $L = \lim a_n = \lim \sqrt{3a_{n-1}} = \sqrt{3L}$

   So $L^2 = 3L$ and thus $L = 0$ or $L = 3$. But a_n is increasing, starting at $a_1 = 1$, so we must have $L = 3$