

Math 1B Quiz 6 SOLUTIONS

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You have 20 minutes to complete this quiz. You must show your work.

1. (3 pts) Find the general solution to $y'' - 4y' + 29y = 0$

Characteristic equation: $r^2 - 4r + 29 = 0$

$$r = \frac{4 \pm \sqrt{16 - 116}}{2} = 2 \pm 5i$$

$$y = C_1 e^{2x} \cos 5x + C_2 e^{2x} \sin 5x$$

2. (4 pts) Solve the initial value problem $y'' + 6y' + 9y = 0$; $y(0) = 1, y'(0) = 1$

Characteristic equation: $r^2 + 6r + 9 = 0$, which factors as $(r + 3)^2 = 0$

General solution is $y = C_1 e^{-3x} + C_2 x e^{-3x}$, so $y' = -3C_1 e^{-3x} + C_2 e^{-3x} - 3C_2 x e^{-3x}$

$y(0) = 1$ gives $1 = C_1 e^0 + C_2 0 e^0 = C_1$

$y'(0) = 1$ gives $1 = -3C_1 e^0 + C_2 e^0 - 3C_2 0 e^0 = -3C_1 + C_2$

So $C_1 = 1, C_2 = 4$ and we get $y = e^{-3x} + 4x e^{-3x}$

3. (3 pts) Re-write each of the given expressions in the indicated form:

(a) $\frac{3 + 4i}{2 - 2i}$ as $a + bi$

$$\frac{3 + 4i}{2 - 2i} = \frac{3 + 4i}{2 - 2i} \frac{2 + 2i}{2 + 2i} = \frac{6 + 8i + 6i - 8}{4 + 4} = -\frac{1}{4} + \frac{7}{4}i$$

(b) $1 + i$ as $r e^{i\theta}$

$a = 1, b = 1$ so $r = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\tan \theta = \frac{1}{1} = 1$, so $\theta = \pi/4$

$$1 + i = \sqrt{2} e^{\frac{\pi}{4}i}$$

(c) i^{30} as $a + bi$

$$i^{30} = i^{28} i^2 = (i^4)^7 (-1) = 1^7 (-1) = -1 = -1 + 0i$$