Name: ______________________________

Math 1B Quiz 5 SOLUTIONS

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You have 30 minutes to complete this quiz. You must show your work.

1. (3 pts) Find the general solution to $xy' - 2y = x^2$

   We divide by x and get $y' - \frac{2}{x}y = x$, which is linear with $p(x) = -\frac{2}{x}, q(x) = x$

$$I(x) = e^{\int -\frac{2}{x}} = e^{-2\ln x} = x^{-2}$$

$$y = \frac{\int IQ}{I} = x^2 \int \frac{1}{x} = x^2(\ln x + C)$$

2. (4 pts) Find the solution to the initial value problem $y' = (2+y)(1+x^2);\ y(0) = 4$.

   This is separable, so we re-write it as $\frac{dy}{2+y} = 1 + x^2 dx$

$$\ln(2+y) = x + \frac{x^3}{3} + C$$

$$2 + y = Ce^{x+x^3/3}$$

   Plugging in $x = 0$ and $y = 4$ gives $6 = Ce^0$, so $C = 6$ and $y = 6e^{x^3/3+x} - 2$

3. (3 pts) Find the general solution to $xy' = x\cos^2\left(\frac{y}{x}\right) + y$. Express your answer in the form $y = ...$

   We divide through by x to get $y' = \cos^2 \frac{y}{x} + \frac{y}{x}$, which leads us to the substitution $v = \frac{y}{x}$. Then $y = vx$, so $y' = v'x + v$ and we have

$$v'x + v = \cos^2 v + v$$

   which reduces to $v' = \frac{\cos^2 v}{x}$

   This is separable and we get

$$\frac{dv}{\cos^2 v} = \frac{dx}{x}$$

$$\tan v = \ln x + C$$

$$v = \tan^{-1}(\ln x + C)$$

   However, since we divided by $\cos^2 v$, we also need to check whether $v = \pi/2 + \pi n$ is a solution to $v' = \cos^2 v/x$, which it is since $\cos^2(\pi/2 + \pi n) = 0$ and $v' = 0$ for any constant function.

   Then since $y = vx$, the general solution is

$$y = x\tan^{-1}(\ln x + C),\ \text{or}\ y = x(\pi/2 + \pi n)\ \text{for any integer n}$$