Name: ______________________________

Math 1B Quiz 3 SOLUTIONS
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You have 20 minutes to complete this quiz. You must show your work. The following fact may be helpful:

If $K > |f''(x)|$ for all $x \in (a,b)$, then $|E_T| \le \frac{K(b-a)^3}{12n^2}$

1. (3 pts) For (a) and (b), write the **form** of the partial fraction decomposition. You need not solve for the coefficients.

   (a) $\frac{1}{(x-2)(x+3)^3} = \frac{A}{x-2} + \frac{B}{x+3} + \frac{C}{(x+3)^2} + \frac{D}{(x+3)^3}$

   (b) $\frac{1}{(x-1)x^2(x^2+x+1)^2} = \frac{A}{x-1} + \frac{B}{x} + \frac{C}{x^2} + \frac{Dx+E}{x^2+x+1} + \frac{Fx+G}{(x^2+x+1)^2}$

   (c) Write $\frac{2x^2-x+1}{x-1}$ in proper form
   Long division gives: $2x+1+\frac{2}{x-1}$

2. (3 pts) Find $\displaystyle\int_0^2 \frac{x}{x^2-1}dx$, or show that it does not exist.

   We note that this is discontinuous at $x=1$, so we write $\int_0^2 \frac{x}{x^2-1}dx = \int_0^1 \frac{x}{x^2-1}dx + \int_1^2 \frac{x}{x^2-1}dx$. Note that $\int \frac{x}{x^2-1}dx = 1/2\ln|x^2-1|$ (if you don't see this, substitute $u = x^2-1$), so

$$\int_0^1 \frac{x}{x^2-1}dx = \lim_{t\to 1^-}\int_0^t \frac{x}{x^2-1}dx = \lim_{t\to 1^-}(\ln|t^2-1| - \ln|-1|) = -\infty$$

   And thus the original integral diverges.

3. (4 pts) Suppose we want to use the Trapezoid rule to estimate $\displaystyle\int_1^2 \frac{1}{x}dx$ and want an error of less than 10^{-5}. How many subintervals (ie, how big of an n) do we need to guarantee this?

   $f(x) = 1/x$, so $f' = -1/x^2$ and $f'' = 2/x^3$. On $(1,2)$, this is always less than 2, so we pick $K=2$

   Then we want

$$\begin{aligned}|E_T| \le \frac{2(2-1)^3}{12n^2} &\le 10^{-5}\\ \frac{10^5}{6} &\le n^2\\ \sqrt{\frac{10^5}{6}} &\le n\end{aligned}$$

   So we can pick any integer greater than $\sqrt{\frac{10^5}{6}}$.