

Name: Solutions

Math 1B Quiz 2  
June 27, 2008  
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You have 20 minutes to complete this quiz. You must show your work.

The following facts may (or may not) be helpful:

$$\sin x \cos x = \frac{1}{2} \sin(2x) \quad \sin(2x) = 2 \sin(x) \cos(x)$$

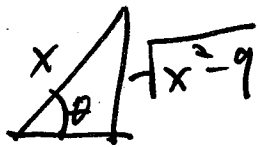
$$\sin^2 x = \frac{1}{2}(1 - \cos(2x)) \quad \cos(2x) = 1 - 2 \sin^2(x)$$

$$\cos^2 x = \frac{1}{2}(1 + \cos(2x)) \quad \cos(2x) = 2 \cos^2(x) - 1$$

$$\tan(2x) = \frac{2 \tan(x)}{1 - \tan^2(x)} \quad \cos(2x) = \cos^2(x) - \sin^2(x)$$

1. (4 pts)  $\int \frac{\sqrt{x^2-9}}{x^3} dx = \int \frac{\sqrt{9(\sec^2\theta-1)}}{27 \sec^3\theta} \cdot 3 \sec\theta \tan\theta d\theta = \frac{1}{3} \int \frac{\tan^2\theta}{\sec^2\theta} d\theta = \frac{1}{3} \int \sin^2\theta d\theta$

$x = 3 \sec\theta$   
 $dx = 3 \sec\theta \tan\theta$



2. (4 pts)  $\int \tan^3\theta \sec\theta d\theta = \int \tan^2\theta \sec\theta \tan\theta d\theta$

$$u = \sec\theta \Rightarrow \int (\sec^2\theta - 1) \sec\theta \tan\theta d\theta$$

$$du = \sec\theta \tan\theta \Rightarrow \int (u^2 - 1) du$$

$$= \frac{u^3}{3} - u + C = \frac{\sec^3\theta}{3} - \sec\theta + C$$

$$= \frac{1}{6} \int (1 - \cos 2\theta) d\theta$$

$$= \frac{\theta}{6} - \frac{\sin 2\theta}{12} + C$$

$$= \frac{\theta}{6} - \frac{\sin\theta \cos\theta}{6} + C$$

$$= \frac{\sec^{-1} \frac{x}{3}}{6} - \frac{\sqrt{x^2-9}}{2x^2} + C$$

3. (2 pts) Complete the square:  $4x^2 + 12x - 12$

$$4x^2 + 12x - 12 = (ax + b)^2 + c$$

$$= a^2x^2 + 2abx + b^2 + c$$

$$a^2 = 4: a = 2$$

$$2ab = 12: b = 3$$

$$b^2 + c = -12: c = -21$$

$$(2x + 3)^2 - 21$$