Name: ______________________________

Math 1B Quiz 11 SOLUTIONS

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You have 30 minutes to complete this quiz. You must show your work.

The following facts may or may not be helpful:

$$f(x) \sim \sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n \qquad R_n(x) = \frac{f^{(n+1)}(z)}{(n+1)!}(x-a)^{n+1} \text{ for some } z \text{ between } a \text{ and } x$$

1. (3 pts) Find a Maclaurin series for $\frac{x}{1-x^2}$. You must express your answer in $\sum$ notation.

$$\frac{x}{1-x^2} = x\frac{1}{1-x^2} = x\sum_{n=0}^{\infty}(x^2)^n = \sum_{n=0}^{\infty} x^{2n+1}$$

2. (a) (4 pts) Find a Taylor series for $f(x) = \sin x$ centered at $\frac{\pi}{2}$. You must express your answer in $\sum$ notation.

n	$f^{(n)}(x)$	$f^{(n)}(\frac{\pi}{2})$
0	$\sin x$	1
1	$\cos x$	0
2	$-\sin x$	-1
3	$-\cos x$	0
4	$\sin x$	1

So the Taylor series is $\sum_{n=0}^{\infty} \frac{(-1)^n(x-\frac{pi}{2})^{2n}}{(2n)!}$

(b) (3 pts) Show that your answer to (a) actually converges to $\sin x$ for all x

From the table above, we see that $f^{(n+1)}(z) = \pm\sin z$ or $\pm\cos z$, depending on the particular value of n. However, no matter what n is, we know $|f^{(n+1)}(z)| \leq 1$ for any z. So we have:

$$0 \leq |R_n(x)| = \left|\frac{f^{(n+1)}(z)(x-\frac{\pi}{2})^{n+1}}{(n+1)!}\right| \leq \left|\frac{(x-\frac{\pi}{2})^{n+1}}{(n+1)!}\right|$$

We know the limit of the left and rightmost parts above are both 0, so by the squeeze theorem, $\lim R_n(x) = 0$ for all x and thus our Taylor series converges to $\sin x$ for all x.