Name: ______________________________

Math 1B Quiz 10 SOLUTIONS
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You have 20 minutes to complete this quiz. You must show your work and clearly **state which tests** you are using.

1. (5 pts) Find the interval and radius of convergence of $\sum_{n=1}^{\infty} \frac{(x-1)^n}{2^n\sqrt{n}}$

   We'll start with the ratio test:

$$\lim_{n\to\infty}\left|\frac{(x-1)^{n+1}}{2^{n+1}\sqrt{n+1}}\frac{2^n\sqrt{n}}{(x-1)^n}\right| = \lim\left|\frac{(x-1)\sqrt{n}}{2\sqrt{n+1}}\right| = \left|\frac{x-1}{2}\right|$$

   So it diverges when $\left|\frac{x-1}{2}\right| > 1$ and converges when $\left|\frac{x-1}{2}\right| < 1$. This is the same as $|x-1| < 2$, so we know it converges when $-1 < x < 3$ and diverges when $x > 3$ or $x < -1$, so we just need to check these endpoints:

   (x=-1): The series becomes $\sum \frac{(-2)^n}{2^n\sqrt{n}} = \sum \frac{(-1)^n}{\sqrt{n}}$ which converges by the alternating series test.

   (x=3): The series is $\sum \frac{2^n}{2^n\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$, which is a divergent p-series ($p = \frac{1}{2}$).

   Thus, the IOC is $[-1, 3)$ and the radius is $\frac{3-(-1)}{2} = 2$

2. (a) (4 pts) Find the interval and radius of convergence for $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

   We'll do the ratio test:

$$\lim_{n\to\infty}\left|\frac{x^{n+1}}{(n+1)!}\frac{n!}{x^n}\right| = \lim\left|\frac{x}{n+1}\right| = 0$$

   so it always converges.

   Thus, $I = (-\infty, \infty)$, $R = \infty$

   (b) (1 pt) Find $\lim_{n\to\infty} \frac{a^n}{n!}$ where a is a constant.

   From part (a), we know $\sum \frac{a^n}{n!}$ converges, so by the test for divergence, we must have $\lim \frac{a^n}{n!} = 0$