

Name: Solutions

Math 1B Quiz 1

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You have 20 minutes to complete this quiz. You must show your work.

1. (3 pts) Find the average value of  $f(x) = x \cos x$  on the interval  $[0, \frac{\pi}{2}]$

$$\begin{aligned}
 &= \frac{1}{\frac{\pi}{2} - 0} \int_0^{\pi/2} x \cos x dx = \frac{2}{\pi} \left( x \sin x \Big|_0^{\pi/2} - \int_0^{\pi/2} \sin x dx \right) = \frac{2}{\pi} \left( \frac{\pi}{2} - 0 + \cos x \Big|_0^{\pi/2} \right) \\
 &\quad \begin{array}{l} u = x \quad dv = \cos x dx \\ du = dx \quad v = \sin x \end{array} \\
 &= \frac{2}{\pi} (0 - 1) \\
 &= \frac{\pi - 2}{\pi}
 \end{aligned}$$

2. (3 pts)  $\int \tan^{-1} x dx = x \tan^{-1} x - \int \frac{x}{1+x^2} dx = x \tan^{-1} x - \frac{1}{2} \int \frac{1}{u} du$

$$\begin{array}{l} u = \tan^{-1} x \quad dv = dx \\ du = \frac{1}{1+x^2} dx \quad v = x \end{array}$$

$$\begin{array}{l} u = 1+x^2 \\ du = 2x dx \end{array}$$

$$\begin{aligned}
 &= x \tan^{-1} x - \frac{1}{2} \ln u + C \\
 &= x \tan^{-1} x - \frac{1}{2} \ln(1+x^2) + C
 \end{aligned}$$

3. (4 pts)  $\int_1^e \frac{\ln(x)}{x^2} dx = -\frac{1}{x} \ln x \Big|_1^e + \int_1^e \frac{1}{x^2} dx = (-\frac{1}{e} + 0) + -\frac{1}{x} \Big|_1^e$

$$\begin{array}{l} u = \ln x \quad dv = \frac{1}{x^2} dx \\ du = \frac{1}{x} dx \quad v = -\frac{1}{x} dx \end{array}$$

$$\begin{aligned}
 &= -\frac{1}{e} - \frac{1}{e} + 1 \\
 &= \frac{e-2}{e}
 \end{aligned}$$