

Math 1B Practice MT2

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The actual midterm will probably be about 6-7 problems, but these should give you an idea of the types of questions to expect.

1. True/False. Justify your answers.

- (a) $y'' + by' + cy = 0$; $y(0) = m, y'(0) = n$ has a solution for any choice of the constants b, c, m, n
- (b) $y'' + by' + cy = 0$; $y(0) = m, y(\pi) = n$ has a solution for any choice of the constants b, c, m, n
- (c) If y_1 and y_2 are solutions to $ay'' + by' + cy = g(x)$, then so is $C_1y_1 + C_2y_2$ for any choice of C_1, C_2
- (d) If y_{p_1} is a solution to $ay'' + by' + cy = g_1(x)$ and y_{p_2} is a solution to $ay'' + by' + cy = g_2(x)$, then $y_{p_1} + y_{p_2}$ is a solution to $ay'' + by' + cy = g_1(x) + g_2(x)$
- (e) If y_{p_1} is a solution to $ay'' + by' + cy = g_1(x)$ and y_{p_2} is a solution to $ay'' + by' + cy = g_2(x)$, then $y_{p_1}y_{p_2}$ is a solution to $ay'' + by' + cy = g_1(x)g_2(x)$

2. Find the general solution to each of the following:

- (a) $\frac{dy}{dx} = (1 + y)^3 \ln(1 + x)$
- (b) $1 + xy = xy'$
- (c) $xy' = y^2/x + 3y + x$
- (d) $y' + \frac{2}{x}y = -\frac{y^3}{x^2}$
- (e) $y'' - 2y' = 1$
- (f) $y'' - 3y' + 2y = \frac{1}{1+e^{-x}}$
- (g) $y'' - y' - 6y = 1 + e^{-2x}$

3. Solve the boundary or initial value problems, or show that no solution is possible

- (a) $y'' + y' - 2y = x + \sin 2x$; $y(0) = 1, y'(0) = 0$
- (b) $y'' + 4y = 0$; $y(0) = 0, y(\pi) = 2$

4. A large tank initially contains 1000L of pure water. Water with 1kg of salt per liter is pumped in at 2L/min. The tank is kept thoroughly mixed and water drains at a rate of 5L/min.

- (a) Write down an initial value problem whose solution would give the amount of salt in the tank at time t
- (b) Solve your equation to find an explicit formula for the amount of salt in the tank at time t
- (c) What is the concentration of salt in the tank right as the last drop drains from the tank?

*Adapted from past midterms from various professors