Math 1B Practice Final

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The actual final will cover the entire course and be 50% integration and ODEs and 50% sequences and series. See previous practice and actual midterms for problems covering material from the first 2/3 of the course.

1. Find the radius and interval of convergence for $\sum_{n=1}^{\infty} \frac{(-2)^n}{\sqrt[3]{n+1}}(x+1)^n$

2. (a) Find the Taylor series for $f(x) = \sin x$, centered at $\frac{\pi}{4}$. In case you've forgotten, $\sin \pi/4 = \cos \pi/4 = 1/\sqrt{2}$
   (b) Find the ROC and IOC for your taylor series from (a).
   (c) Prove that your taylor series converges to $\sin x$ for all x.

3. Find the solution to the initial value problem $y'' - xy' - y = 0$; $y(0) = 1, y'(0) = 0$. Your answer may involve power series if you wish.

4. True/False. Justify your answers with theorems/tests or counterexamples.
   (a) If $\sum c_n x^n$ has radius of convergence R, then $\sum c_n R^n$ converges conditionally.
   (b) If $\sum c_n 3^n$ converges, then $\sum (-1)^n c_n 3^n$ converges.
   (c) If $\sum c_n 3^n$ converges, then $\sum (-1)^n c_n 2^n$ converges.
   (d) If $\sum a_n$ and $\sum b_n$ are conditionally convergent, then $\sum a_n + b_n$ is too
   (e) If $\sum a_n$ is absolutely convergent, then $\sum a_n^2$ is too

5. Determine whether each of the following series are absolutely convergent, conditionally convergent, or divergent:
   (a) $\sum \frac{\tan^{-1} n}{n^3}$
   (b) $\sum \frac{n!}{e^{n^2}}$
   (c) $\sum \frac{n!}{2 \cdot 5 \cdot 8 \cdots (3n+2)}$
   (d) $\sum \frac{(-1)^n}{n\sqrt{\ln n}}$
   (e) $\sum \frac{1}{1+e^n}$
   (f) $\sum \left(\frac{1}{n^{5/7}} - \frac{1}{(n+1)^{5/7}} \right)$
   (g) $\sum \frac{\cos(\pi n)}{n(\ln n)^2}$
   (h) $\sum (-1)^n \frac{3^n}{n^3}$
   (i) $\sum \frac{(-2)^{3n}}{n^n}$
   (j) $\sum \ln \frac{n+2}{n}$

*Adapted from past midterms from various professors

(k) $\sum \dfrac{\sqrt{n+1}-\sqrt{n}}{n}$

(l) $\sum (-1)^n \sqrt[n]{2}$

6. Using whatever means you want, find the Maclaurin series for each of the following functions:

(a) $\frac{\ln(1+x)}{x}$

(b) $\frac{x}{1+x^3}$

(c) $\int e^{x^2}$

(d) $\frac{x^2}{3+x}$