

Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Find a formal solution to the vibrating string problem

$$\begin{aligned}u_{tt} &= 4u_{xx} + x^2 \\u(0, t) = u(1, t) &= 0 \\u(x, 0) &= 0 \\u_x(x, 0) &= x\end{aligned}$$

2. Let $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} \end{bmatrix}$. Find A^{-1}

3. Without using the methods of section 9.5, solve the system of differential equations $\mathbf{y}' = \begin{bmatrix} 0 & 1 \\ -1 & -2 \end{bmatrix} \mathbf{y}$
4. Find all eigenvalues and eigenvectors of $L[y] := y'' + 2y'$ over the vector space $V = \{f \in C^\infty : f(0) = f(2\pi) = 0\}$
5. Consider \mathbb{P}_3 with the inner product $\langle p, q \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1)$. Find the linear function f that minimizes $\|f - t^3\|$
6. Find the Fourier sine series of $f(x) = |x|$ on $(0, 1)$