

Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Let $f(x) = L$ on $(0, L)$
 - (a) Draw the even $2L$ -periodic extension of f and compute its cosine series.
 - (b) Draw the odd $2L$ -periodic extension and compute its sine series.
2.
 - (a) Compute the Fourier sine series of $f(x) = x^2, 0 < x < \pi$
 - (b) Using your answer from (a), find the solution to the heat flow problem with $\beta = 2, L = \pi,$
 $u(0, t) = u(\pi, t) = 0, u(x, 0) = x^2$
3. For each of the following, use the method of separation of variables to reduce the given PDE to 2 ordinary differential equations or explain where the method fails. DO NOT solve the resulting system of equations unless you get really bored this weekend.
 - (a) $xu_{xx} = u_t$
 - (b) $xu_{xx} + (x + t)u_t = 0$
 - (c) $tu_{xx} + xu_t = 0$
 - (d) $u_{xx} + \frac{1}{x}u_x + \frac{1}{x^2}u_{tt} = 0$
4. Consider the PDE $u_t + tu = u_{xx}$ with boundary conditions $u_x(0, t) = u_x(\pi, t) = 0$.
 - (a) Use the method of separation of variables to find all solutions of the form $u(x, t) = X(x)T(t)$.
 - (b) Find a solution satisfying the initial condition $u(x, 0) = \sin^2 x$