

# Math 54 Discussion Section Problems

Rob Bayer

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Compute the Fourier series for  $f(x) = |x|$  on the interval  $(-\pi, \pi)$ .
2. Compute the Fourier series for  $f(x) = \begin{cases} x & 0 < x < 1 \\ 0 & -1 < x < 0 \end{cases}$  on
3. Solve the heat equation  $u_t = 100u_{xx}$  with initial conditions  $u(0, t) = u(1, t) = 0$  and  $u(x, 0) = \sin 2\pi x - 2 \sin 5\pi x$
4. For each of the following, use the method of separation of variables to reduce the given PDE to 2 ordinary differential equations or explain where the method fails. DO NOT solve the resulting system of equations unless you get really bored this weekend.
  - (a)  $xu_{xx} = u_t$
  - (b)  $xu_{xx} + (x+t)u_t = 0$
  - (c)  $tu_{xx} + xu_t = 0$
  - (d)  $u_{xx} + \frac{1}{x}u_x + \frac{1}{x^2}u_{tt} = 0$
5. Consider the PDE  $u_t + tu = u_{xx}$  with boundary conditions  $u_x(0, t) = u_x(\pi, t) = 0$ .
  - (a) Use the method of separation of variables to find all solutions of the form  $u(x, t) = X(x)T(t)$ .
  - (b) Find a solution satisfying the initial condition  $u(x, 0) = \sin^2 x$