Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Suppose that $\mathbf{y}' = A\mathbf{y}$ is a system of differential equations in normal form and that A is composed entirely of constants. Show that if $\mathbf{u}$ is an eigenvector of A with eigenvalue λ, then $\mathbf{y} = e^{\lambda t}\mathbf{u}$ is a solution to the system of differential equations.
2. Let $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$. Solve the system of equations $\mathbf{y}' = D\mathbf{y}$
3. Find the general solution to the first order system of equations $\mathbf{x}' = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 2 & 1 & 2 \end{bmatrix} \mathbf{x}$
4. Let $A = \begin{bmatrix} 1 & -1 \\ 4 & -3 \end{bmatrix}$ and consider the system of equations $\mathbf{x}' = A\mathbf{x}$
   (a) Find the eigenvalues and eigenvectors of A. You should only be able to find one linearly independent eigenvector.
   (b) Let $\mathbf{x}_1$ be the solution to $\mathbf{x}' = A\mathbf{x}$ you get from your answer in part (a). To find a second solution, we'll guess $\mathbf{x}_2 = te^{-t}\mathbf{u}_1 + e^{-t}\mathbf{u}_2$. Plug $\mathbf{x}_2$ into the equations $\mathbf{x}' = A\mathbf{x}$ and show that we must have $(A + I)\mathbf{u}_1 = 0, (A + I)\mathbf{u}_2 = \mathbf{u}_1$.
   (c) Solve for $\mathbf{u}_1, \mathbf{u}_2$. Hint: you should have already found a vector that works for $\mathbf{u}_1$.
   (d) Write down and simplify the general solution: $\mathbf{x} = C_1\mathbf{x}_1 + C_2\mathbf{x}_2$
   (e) What is $(A + I)^2\mathbf{u}_2$? Note: this is not a coincidence. $\mathbf{u}_2$ is called a generalized eigenvector and you are always guaranteed to be able to find them for repeated eigenvalues with not enough eigenvectors.