

# Math 54 Discussion Section Problems

Rob Bayer

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Express the second order equation  $y'' + xy' - \sqrt{x}y = 0$  as a system of first order equations in normal form.

2. Re-write the system of equations 
$$\begin{cases} x'_1 = 3x_1 + \sin tx_2 + e^t x_3 + \cos t \\ x'_2 = -x_2 + tx_3 \\ x'_3 = -\sqrt{t}x_1 + x_3 + e^t \end{cases}$$

in matrix normal form notation.

3. Now combine what you did for questions 1 and 2 to re-write the system of higher order equations

$$\begin{aligned} x'' - 3x' + t^2y - (\cos t)x &= 0 \\ y'' - tx' + y' + e^t x &= 0 \end{aligned}$$

as a matrix system in normal form. Note here that  $y, x$  are both functions of  $t$ .

4. True/False. If true, prove it. If false, provide a counterexample.

If  $a, b, c, x_0, x_1, y_0, y_1$  are all constants, then  $ay'' + by' + cy = 0; y(x_0) = y_0, y(x_1) = y_1$  always has a solution. (ie, boundary value problems for equations with constant coefficients are always solvable)

5. Suppose that  $\mathbf{y}' = A\mathbf{y}$  is a system of differential equations in normal form and that  $A$  is composed entirely of constants. Show that if  $\mathbf{u}$  is an eigenvector of  $A$  with eigenvalue  $\lambda$ , then  $\mathbf{y} = e^{\lambda t}\mathbf{u}$  is a solution to the system of differential equations.