Math 54 Discussion Section Problems

Rob Bayer

April 18, 2008

You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Find all roots to $r^3 - 3r^2 + 7r - 5$, and $r^4 - 7r^3 + 16r^2 - 12r$.
2. Use the Wronskian to show that each of the following sets of functions are linearly independent on any open interval (a, b):
   (a) e^x, e^{2x}, e^{-1}
   (b) $x, e^x, 1$
3. Solve the differential equation $xy'' - (2x^2 + 1)y' = 0$ on $(0, \infty)$ by making the change of variables $z = y'$
4. For this entire problem, you only need to solve the differential equation on $(1, \infty)$
   (a) Show that $\{x, x^2, x^3\}$ form a fundamental set of solution to $x^3y''' - 3x^2y'' + 6xy' - 6y = 0$
   (b) Use the method of undetermined coefficients to find the general solution to $x^3y''' - 3x^2y'' + 6xy' - 6y = 18$.
   (c) Now solve the initial value problem $x^3y''' - 3x^2y'' + 6xy' - 6y = 18; y(1) = 3; y'(1) = 14; y''(1) = 24$
   (d) Can you solve the initial value problem $y(0) = 1; y'(0) = 2; y''(0) = 3$? Why does this not contradict our theorem about existence and uniqueness of solutions?
5. Hint for the below problems: you should probably do 3 cases for each part, one for each of the possible number of roots of the characteristic equation.
   (a) Show that if a, b, c are all greater than 0, then all solutions to $ay'' + by' + cy = 0$ have the property that $\lim_{x\to\infty} y(x) = 0$
   (b) If $a > 0, c > 0$ but $b = 0$, show that the result from part (a) is no longer true, but that all solutions are bounded as $x \to \infty$
   (c) If $a > 0, b > 0$ and $c = 0$, show that all solutions approach some constant (not necessarily 0) as $x \to \infty$. Determine this constant in terms of $y(0)$, which we'll call y_0, and $y'(0)$, which we'll call y_0'