

Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

- Which of the following second order differential equations are linear? Homogeneous?
 - $e^x y'' + \cos(3x^2)y' + 3y = 0$
 - $y'' + 3y' + 7y = \cos x$
 - $y'' + 3y' + y^2 = 0$
 - $\tan(y'') + \cos(x)y' = e^x$
- Find the general solution to $y'' - 4y' + 4y = 0$
- Solve the initial value problem $y'' + 3y' - 10y = 0$ with $y(0) = 1$ and $y'(0) = 3$
- Solve the boundary value problem $y'' = y$ with $y(0) = 0$ and $y(2) = 2$
- In the book and in lecture, we saw that two solutions to $ay'' + by' + cy = 0$ are linearly dependent if and only if there is some $\tau \in I$ such that $W(y_1, y_2)(\tau) = 0$. It turns out that we can even extend this to functions which are not solutions to a differential equation by changing the theorem to: f, g are linearly **independent** on I if and only if $W(f, g)(t) \neq 0$ for **some** $t \in I$. Using this, determine whether each of the following pairs of functions are linearly independent or dependent:
 - $f(x) = x, g(x) = x^2$
 - $f(x) = e^x, g(x) = e^{-x}$
 - $f(x) = e^x, g(x) = e^{x+1}$
 - $f(x) = xe^{2x}, g(x) = e^{2x}$
 - $f(x) = \ln(x^3), g(x) = \ln(x^\pi)$
 - $f(x) = \sin(x), g(x) = \cos(x)$