Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. For each of the following, determine whether the given rule determines an inner product on the vector space. For those that do, prove it. For those that don't, show which axiom it violates. C^0 is the space of all continuous functions.
   (a) $V = C^0, \langle f, g\rangle = \int_0^{2\pi} f(x)g(x)dx$
   (b) $V = C^0, \langle f, g\rangle = f(0)g(0)$
   (c) $V = \mathbb{P}_2, \langle p, q\rangle = p(-2)q(-2) + p(1)q(1) + p(\pi)q(\pi)$
   (d) $V = \mathbb{R}^2, \langle \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}, \begin{bmatrix} v_1 \\ v_2 \end{bmatrix} \rangle = 2u_1v_1 + 7u_2v_2$

2. Let V be the inner product space from part (c) above and let $p(t) = 2 + t, q(t) = 3 + t - t^2$
   (a) Compute $\langle p, q\rangle$
   (b) Find $\|p\|$
   (c) Find a non-zero polynomial orthogonal to p. Hint: if you think about this correctly, you shouldn't have to do much work.

3. In this problem, we'll try to find the line $y = mx + b$ that "best fits" the points (-1, -1), (1,0), (2,4):
   (a) Write down the system of equations that would need to be satisfied for these three points to line on the line $y = mx + b$ and verify that this system is inconsistent. (Hint: your variables should be m, b)
   (b) Re-write this as a matrix equation
   (c) Use the least squares method to find the best possible m, b
   (d) Now plot these three points and the line you found. Does your answer look reasonable?

4. Let $V = C[-1, 1]$, the continuous functions on [-1,1]. Let $\langle f, g\rangle = \int_{-1}^{1} f(x)g(x)dx$, and $W = \text{span}\ \{1, t,^2\}$
   (a) Find an orthogonal basis for W
   (b) Find the best quadratic polynomial approximation to $\sin t$ with respect to this inner product.
   (c) Sketch your approximation and $\sin t$. Does your answer look reasonable?

5. Prove that $\langle \mathbf{u}, \mathbf{v}\rangle = \frac{1}{4}\|\mathbf{u} + \mathbf{v}\| - \frac{1}{4}\|\mathbf{u} - \mathbf{v}\|$ for any inner product and vectors.

6. Prove that $\mathbf{y} - \text{Proj}_W\mathbf{y} \in W^{\perp}$. In other words, prove that our method of finding $\widehat{\mathbf{y}}$ and $\mathbf{z}$ really works.