Math 54 Discussion Section Problems

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April 2, 2008

You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Find a QR-factorization for $\begin{bmatrix} 1 & 2 & 5 \\ -1 & 1 & -4 \\ -1 & 4 & -3 \\ 1 & -4 & 7 \\ 1 & 2 & 1 \end{bmatrix}$

2. Find all least squares solutions to $A\mathbf{x} = \mathbf{b}$, where $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 1 \\ 3 \\ 8 \\ 2 \end{bmatrix}$

3. In this problem, we'll try to find the line $y = mx + b$ that "best fits" the points (-1, -1), (1,0), (2,4):
   (a) Write down the system of equations that would need to be satisfied for these three points to line on the line $y = mx + b$ and verify that this system is inconsistent. (Hint: your variables should be m, b)
   (b) Re-write this as a matrix equation
   (c) Use the least squares method to find the best possible m, b
   (d) Now plot these three points and the line you found. Does your answer look reasonable?

4. Suppose that $A = QR$, where R is an $n \times n$ matrix. Prove that if the columns of A are linearly independent, then R must be invertible. (Hint: the columns of A are linearly independent iff $Ax = 0$ has only the trivial solution.)

5. Prove that $\mathbf{y} - \text{Proj}_W \mathbf{y} \in W^\perp$. In other words, prove that our method of finding $\widehat{\mathbf{y}}$ and $\mathbf{z}$ really works.