# Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ Find each of the following:
   (a) $\mathbf{u} \cdot \mathbf{v}$
   (b) $\|\mathbf{u}\|$
   (c) The distance between $\mathbf{u}$ and $\mathbf{v}$
   (d) $\mathbf{u}\mathbf{v}^T$
2. Is the set $\{(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}, 0), (0, \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}), (\frac{1}{2}, \frac{1}{2}, -\frac{1}{2}, -\frac{1}{2})\}$ orthonormal?
3. Let $W$ be a subspace of a vector space $V$ with an inner product. Prove that $W^\perp$ is also a subspace of $V$
4. Suppose $W$ =span$\{(1, 1, 1), (2, 0, -1)\}$. Find a basis for $W^\perp$
5. Show that for any real number $\theta$, the set $\{(\cos\theta, \sin\theta), (-\sin\theta, \cos\theta)\}$ is an orthonormal basis for $\mathbb{R}^2$
6. Let $\mathbf{y} = \begin{bmatrix} 4 \\ 3 \\ 3 \\ -1 \end{bmatrix}, \mathbf{u}_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \mathbf{u}_2 = \begin{bmatrix} -1 \\ 3 \\ 1 \\ -2 \end{bmatrix}, \mathbf{u}_3 = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \end{bmatrix}$ and $W = \text{span}(\mathbf{u}_1, \mathbf{u}_2, \mathbf{u}_3)$
   (a) Verify that the $\mathbf{u}$'s are an orthogonal set
   (b) Write $\mathbf{y}$ as as $\widehat{\mathbf{y}} + \mathbf{z}$ where $\widehat{\mathbf{y}} \in W, \mathbf{z} \in W^\perp$
   (c) Find the distance from $\mathbf{y}$ to $W$
7. (a) Let $\mathbf{x}, \mathbf{u} \in V$ with $\mathbf{u} \neq \mathbf{0}$. Prove that $\mathbf{x} - \text{Proj}_{\mathbf{u}}\mathbf{x}$ is orthogonal to $\mathbf{u}$
   (b) Let $\mathbf{y} \in V$ and let $\mathbf{u}_1, \mathbf{u_2}, \ldots, \mathbf{u}_p$ be an orthogonal basis for $W$. Prove that $\mathbf{y} - \text{Proj}_W\mathbf{y} \in W^\perp$
8. Let $W$ be a subspace of $\mathbb{R}^n$. Prove that if $\mathbf{v} \in W$ and $\mathbf{v} \in W^\perp$, then $\mathbf{v} = \mathbf{0}$
9. True/False. Justify your answers.
   (a) If $\mathbf{u} \cdot \mathbf{v} = 0$, then either $\mathbf{u}$ or $\mathbf{v} = 0$
   (b) If $\mathbf{u} \neq 0$, then $\frac{u}{\|u\|}$ is a unit vector
   (c) $\mathbf{0}$ cannot be in any orthogonal set