Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Find the characteristic polynomial of $\begin{bmatrix} 5 & 6 & 2 \\ 0 & -1 & -8 \\ 1 & 0 & 2 \end{bmatrix}$

2. Find the eigenvalues and bases for the associated eigenspeaces of $\begin{bmatrix} 2 & 1 \\ -1 & 4 \end{bmatrix}$

3. Suppose A is a 4×4 matrix with eigenvalues $-1, 0, 2$. From this information, do you know $\det A$? If yes, calculate it. If not, give examples of 2 matrices with these eigenvalues but different determinants.

4. Let $V = C^\infty$, the space of all infinitely differentiable functions and let $T : C^\infty \to C^\infty$ be the linear transformation defined by $T(f) = f'$. Using the definition of eigenvalue/eigenvector (don't worry, it's the same for linear transformations as for matrices), find the eigenvalues and associated eigenspaces of T

   (Hint: You will need to solve a differential equation to do this problem. This is intentional and should be a good refresher.)

5. True/False. Justify your answers by either proving the given statement or providing a counterexample.
   (a) $\mathbf{0}$ is an eigenvector of every matrix.
   (b) If a matrix has at least one eigenvector, then it has infinitely many eigenvectors.
   (c) Every square matrix has at least 1 real eigenvalue.
   (d) The sum of two eigenvalues of A is an eigenvalue of A
   (e) The sum of two eigenvectors of A is an eigenvector of A.

6. For each of the following linear transformations, describe the eigenspaces (if any) and their associated eigenvalues. Do **not** compute the matrices–you should do this from a purely geometric viewpoint.
   (a) $T : \mathbb{R}^3 \to \mathbb{R}^3$, where T reflects vectors across the xy-plane
   (b) $T : \mathbb{R}^2 \to \mathbb{R}^2$, where T rotates vectors by $90°$ CCW.