

Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

- (Tataru, Spring 08) Find $\det \begin{bmatrix} 7 & 0 & 0 & 4 & 0 \\ 1 & 1 & 2 & 5 & 0 \\ 1 & 4 & 7 & 5 & 2 \\ 3 & 0 & 0 & 2 & 0 \\ 2 & 0 & 1 & 1 & 0 \end{bmatrix}$
- (Tataru, Spring 08) Let $A = \begin{bmatrix} 0 & 1 & 2 & 3 \\ 2 & 1 & 1 & 2 \\ 4 & 1 & 0 & 1 \end{bmatrix}$. Find Rank A and bases for Col A , Row A , Nul A
- (Gu, Fall 07) Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 1 & 2 & k^2 \\ 2 & 1 & 1 \end{bmatrix}$, and $b = \begin{bmatrix} 6 \\ 3k \\ 4 \end{bmatrix}$. For what value(s) of k does $Ax = b$ have a unique solution? Infinitely many solutions? No solutions?
- (Gu, Fall 07) Find $n \times n$ matrices A, B , such that neither A nor B are invertible, but $A + B$ is.
- Find the area of the parallelogram defined by $\begin{bmatrix} -2 \\ 3 \end{bmatrix}$, $\begin{bmatrix} -2 \\ 5 \end{bmatrix}$
- True/False. Justify your answers:
 - Nul $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is \mathbb{R}^2
 - (Gu, Fall 07) $((AB)^{-1})^T = (A^{-1})^T(B^{-1})^T$
 - If A is a $n \times n$ matrix, then $\det(2A) = 2 \det A$
 - The map $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $T \left(\begin{bmatrix} x \\ y \end{bmatrix} \right) = \begin{bmatrix} x + y - 1 \\ x + 2y + 1 \end{bmatrix}$ is a linear transformation. (Hint: there are many ways to do this problem. See if you can come up with a one-liner)
 - If A and B are matrices such that $AB = 0$, then either A or B must be 0.
 - If A and B are $n \times n$ matrices such that AB is invertible, then BA is invertible.
- Let $B = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}$, and let W be the set of all 2×2 matrices A such that $AB = 0$.
 - Show that W is a subspace of $M_{2 \times 2}$.
 - Find a basis for W . (This is a hard problem. If you don't get it, don't feel bad; it's meant to be a challenge)
 - Find $\dim W$