

# Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Let  $\mathcal{B} = \{1 + t^2, 1 + t, 1 + 2t + t^2\}$  and  $\mathcal{C} = \{1 + 2t, 3 + t^2, 2 - 2t + 3t^2\}$ . We have previously shown that both  $\mathcal{B}$  and  $\mathcal{C}$  are a bases for  $\mathbb{P}_2$ . Let  $\mathcal{S} = \{1, t, t^2\}$ , (ie, the standard basis for  $\mathbb{P}_2$

(a) Find  $\mathcal{P}_{\mathcal{S} \leftarrow \mathcal{B}}$

(b) Find  $\mathcal{P}_{\mathcal{C} \leftarrow \mathcal{S}}$

(c) Use your answers to parts (a) and (b) to find  $\mathcal{P}_{\mathcal{C} \leftarrow \mathcal{B}}$

2. Let  $\mathcal{B} = \left\{ \begin{bmatrix} 7 \\ 5 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \end{bmatrix} \right\}, \mathcal{C} = \left\{ \begin{bmatrix} 1 \\ -5 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix} \right\}$ . Find  $\mathcal{P}_{\mathcal{B} \leftarrow \mathcal{C}}$  and  $\mathcal{P}_{\mathcal{C} \leftarrow \mathcal{B}}$

3. Suppose you have some vector space  $V$ , a basis  $\mathcal{B} = \{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$  and vector  $\mathbf{x}$  with  $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}$ .

Now suppose that someone gives you a new set of vectors  $\mathcal{C} = \{\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3\}$  and tells you that  $[\mathbf{c}_1]_{\mathcal{B}} =$

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, [\mathbf{c}_2]_{\mathcal{B}} = \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}, [\mathbf{c}_3]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}.$$

(a) Is  $\mathcal{C}$  a basis for  $V$ ?

(b) Find the change of coordinate matrices from (i)  $\mathcal{C}$  to  $\mathcal{B}$  and (ii)  $\mathcal{B}$  to  $\mathcal{C}$

(c) Find  $[\mathbf{x}]_{\mathcal{C}}$