Math 54 Discussion Section Problems

Rob Bayer

February 29, 2008

You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Suppose you have some vector space V, a basis $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$ and vector $\mathbf{x}$ with $[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}$.

   Now suppose that someone gives you a new set of vectors $\mathcal{C} = \{\mathbf{c_1}, \mathbf{c_2}, \mathbf{c_3}\}$ and tells you that $[\mathbf{c_1}]_{\mathcal{B}} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}$, $[\mathbf{c_2}]_{\mathcal{B}} = \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}$, $[\mathbf{c_3}]_{\mathcal{B}} = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$.

   (a) Is $\mathcal{C}$ a basis for V?

   (b) Find $[\mathbf{x}]_{\mathcal{C}}$

   (c) Find the change of coordinate matrices from (i) $\mathcal{C}$ to $\mathcal{B}$ and (ii) $\mathcal{B}$ to $\mathcal{C}$

2. Let A be some fixed 3×4 matrix. Prove that the set $\{B \in M_{4\times 2} : AB = \mathbf{0}\}$ is a subspace of $M_{4\times 2}$.

3. Show that if A is a 30×36 matrix and $\dim \operatorname{Nul} A = 6$, then $A\mathbf{x} = \mathbf{b}$ has a solution for every $\mathbf{b} \in \mathbb{R}^{30}$

4. Find a basis for the row space, the column space, and the null space of $A = \begin{bmatrix} 1 & -4 & 9 & -7 \\ -1 & 2 & -4 & 1 \\ 5 & -6 & 10 & 7 \end{bmatrix}$, which is row equivalent to $\begin{bmatrix} 1 & 0 & -1 & 5 \\ 0 & -2 & 5 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

5. Theorem 11 from section 4.5 of the book tells us that given any linearly independent vectors $\{\mathbf{v_1}, \mathbf{v_2}, \dots \mathbf{v_p}\}$ in $\mathbb{R}^n$, we can extend it to a basis for $\mathbb{R}^n$ by adding $n-p$ vectors to it. However, it doesn't really give us an idea of how to actually do this. Here's one possible way: create the matrix $A = [\mathbf{v_1}\mathbf{v_2}\cdots\mathbf{v_p}\mathbf{e_1}\cdots\mathbf{e_n}]$ ($\mathbf{e_i}$ are the columns of the identity matrix) and then use methods we already know to find a basis for Col A.

   (a) How do you know the basis you come up with will be a basis for $\mathbb{R}^n$?

   (b) How do you know it will include the vectors $\mathbf{v_1}, \mathbf{v_2}, \dots \mathbf{v_p}$?

   (c) Use this method to extend the following set of vectors to be a basis for $\mathbb{R}^4$:

   $$\mathbf{v_1} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 3 \\ 8 \\ -2 \\ 3 \end{bmatrix}$$