Math 54 Discussion Section Problems

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Find the β-coordinate vector for $p(t) = 3 - 8t + 4t^2$ where $\beta = \{1 + t^2, t + t^2, 1 + 2t + t^2\}$
2. Suppose you have some vector space V, a basis $\beta = \mathbf{b_1}, \mathbf{b_2}, \mathbf{b_3}\}$ and vector $\mathbf{x}$ with $[\mathbf{x}]_\beta = \begin{bmatrix} 8 \\ -9 \\ 6 \end{bmatrix}$. Now suppose that someone gives you a new set of vectors $\Gamma = \{\mathbf{c_1}, \mathbf{c_2}, \mathbf{c_3}\}$ and tells you that $[\mathbf{c_1}]_\beta = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, [\mathbf{c_2}]_\beta = \begin{bmatrix} -3 \\ 4 \\ 9 \end{bmatrix}, [\mathbf{c_3}]_\beta = \begin{bmatrix} 2 \\ -2 \\ 4 \end{bmatrix}$.
   (a) Is Γ a basis for V?
   (b) Find $[\mathbf{x}]_\Gamma$
   (c) Find the change of coordinate matrices from (i) Γ to β and (ii) β to Γ
3. Consider the transformation $T : \mathbb{P}_3 \to \mathbb{P}_3$ given by $T(p) = \frac{d}{dx}p$. Prove that T is a linear transformation and determine if it is 1-1 and if it is onto.
4. Let A be some fixed 3×4 matrix. Prove that the set $\{B \in M_{4\times 2} : AB = 0\}$ is a subspace of $M_{4\times 2}$.
5. Determine whether each of the following are subspaces of $\mathbb{P}_3$[1]
   (a) $\{p \in \mathbb{P}_3 : p(1) = 0\}$
   (b) $\{p \in \mathbb{P}_3 : p(1) = 1\}$
   (c) The set of all odd functions in $\mathbb{P}_3$
6. Do the invertible 3×3 matrices form a subspace of $M_{3\times 3}$?[2] What about the matrices of the form $\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$? Of the form $\begin{bmatrix} a & b & c \\ 1 & d & e \\ 1 & 1 & f \end{bmatrix}$?

[1] $\mathbb{P}_n$ denotes the set of all polynomials with degree less than or equal to n
[2] $M_{m\times n}$ denotes the set of all $m \times n$ matrices