# Math 54 Discussion Section Problems

Rob Bayer

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You should work on the following problems in groups of 3 or 4. Try to get through as many as you can, but you aren't expected to finish everything. In fact, the answers are largely unimportant; making sure **everyone** in your group knows **how** to solve all the problems is what really matters.

1. Determine which of the following are vector spaces. For those that are, prove it and determine what $\nvdash$ is. For those that aren't, show which axiom it violates:
   (a) The set of all polynomials of degree at most 2
   (b) The set of all polynomials of degree exactly 2
   (c) The set of all infinite sequences of real numbers
   (d) The set of all power series
   (e) The set of all $2 \times 3$ matrices
   (f) The set of all continuous functions from $\mathbb{R}$ to $\mathbb{R}$
2. Determine whether each of the following are subspaces of $\mathbb{P}_3$[1]
   (a) $\{p \in \mathbb{P}_3 : p(1) = 0\}$
   (b) $\{p \in \mathbb{P}_3 : p(1) = 1\}$
   (c) The set of all odd functions in $\mathbb{P}_3$
3. Do the invertible $3 \times 3$ matrices form a subspace of $M_{3\times3}$?[2] What about the matrices of the form $\begin{bmatrix} a & b & c \\ 0 & d & e \\ 0 & 0 & f \end{bmatrix}$? Of the form $\begin{bmatrix} a & b & c \\ 1 & d & e \\ 1 & 1 & f \end{bmatrix}$?
4. Consider the transformation $T : \mathbb{P}_3 \to \mathbb{P}_3$ given by $T(p) = p'$. Prove that $T$ is a linear transformation and determine if it is 1-1 and if it is onto.
5. Let $A$ be some fixed $2 \times 3$ matrix. Prove that the set $\{B \in M_{4\times2} : AB = 0\}$ is a subspace of $M_{4\times2}$.

[1] $\mathbb{P}_n$ denotes the set of all polynomials with degree less than or equal to $n$

[2] $M_{m\times n}$ denotes the set of all $m \times n$ matrices